Frequency Response Function of a Truss Spar Subjected to Wave Loads

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***Abstract*—As sea state conditions vary from one oceanic region to another, no two similar structures will behave similarly when installed in different oceanic regions. Through measurements of structural responses and metocean loading in a particular location, the frequency response function, which describes the behavior of the structure given any metocean loading from any geographical location, can be determined. In this paper, a truss spar model is developed based on a scale of 1:100 and is tested in a wave tank to measure its responses due to simulated random wave loads. The frequency response functions are then determined using the time histories of measured structural response and wave height.**

Keywords—frequency response function; truss spar; floating structure; power spectrum; wave loads

I. INTRODUCTION

A truss spar shown in Fig. 1 [1] is a floating structure that is usually installed in deepwater environments. It is held in place by mooring lines that are attached to the spar and tied to the seabed.

Fig. 1. A truss spar

As wind, wave and current loads are applied onto the structure, the spar can respond in six degrees of freedom (DOFs) corresponding to three translational movements and three rotational movements, as shown in Fig. 2.

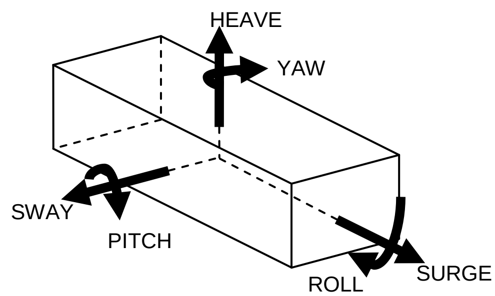

Fig. 2. The six degrees of freedom of a floating structure

Two similar structures will behave differently in two different oceanic regions. If the frequency response function (FRF) of a floating structure is known, then the response and behavior of the structure can be determined for any metocean conditions. Besides enabling the possibility of employing the same structural design for a different location, the FRF can help to deduce information on the survivability of the structure prior to a major event such as hurricanes or tsunamis. Through measured sea state data and structural response data, the FRF can be determined.

In this paper, a framework is outlined to obtain the FRF of a truss spar model which is developed to a scale of 1:100 and subjected to simulated random waves produced by wave generators in a wave tank; the wave heights and structural responses in all DOFs are measured and recorded.

II. LITERATURE REVIEW

A. Frequency Response Function

A time series is a sequence of observations measured sequentially in time with constant intervals between two adjacent observations [2]. As measured structural response data and sea state data represent stochastic processes, these time histories are analyzed statistically to understand the behavior of the measured processes.

One such statistical measure is the autocorrelation function which portrays the linear relationship between the time series $x(t)$ and a lagged version of itself $x(t + \tau)$, where τ is the lag number [2]. If the autocorrelation returns a value of unity, it indicates that the time series *x(t)* and *x(t + τ)* are perfectly correlated. If they are uncorrelated, the autocorrelation function returns a value of zero. For a weakly stationary mean-zero random process, the autocorrelation function is a function of lag τ expressed as [3, 4]:

$$R_{XX}(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u\, v\, p_{X(t)X(t+\tau)}(u,v)\, du\, dv \quad (1)$$

where $p_{X(t)X(t+\tau)}(u,v)$ is the probability distribution function of the random process $\{X(t)\}$.

The power spectral density function decomposes a stochastic process into its frequency components to identify its periodicities in the process. For a non-zero mean random process, the power spectral density function is determined by applying the Fourier transform on the autocovariance of the time series. For a weakly stationary zero-mean random process, the power spectral density function can be obtained by applying the Fourier transform of the autocorrelation function directly. If a time series has non-zero mean, the mean can be removed before applying the Fourier transform on the autocorrelation function. The power spectral density function is given by [4]:

$$S_{XX}(f) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-i2\pi f\tau} d\tau \quad (2)$$

where *i* is the imaginary number defined as the square root of -1.

A linear, time-invariant system with excitation *F(f)* and response *X(f)* expressed in the frequency domain can be described by the frequency response function, where the relationship is given by [3]:

$$S_{XX}(f) = |H_x(f)|^2 S_{FF} \quad (3)$$

where $S_{XX}(f)$ is the power spectrum of the structural response, $S_{FF}(f)$ is the power spectrum of the excitation, and $H_X(f)$ is the FRF. It is clear that through recording the time histories of the excitation and those corresponding to their respective structural response, the FRF can be determined.

B. Sea Spectra Application for Wave Generation

In wave tank tests, wave parameters are modeled after existing sea spectra that are established for offshore structural design. This method is suitable for wave generation as it can simulate random conditions of actual seas where the waves are thought to be constructed as a finite sum of Fourier waves. In the frequency domain, this would be represented as a distribution of spectral density over a range of wave frequencies. Alternatively, regular waves can be generated and applied on modeled offshore structures, although such waves are highly unrepresentative of actual sea wave states [5].

An assumption for the wave tank test is to replicate the sea states to that of a fetch limited sea. This indicates limitation in the assumptions of two parameters, namely fetch length and fetch duration. Therefore, the Joint North Sea Wave Project (JONSWAP) spectrum is employed to model the required sea states because it was designed with the assumption of fetch limitation taken into account [6, 7]. The characteristic of fetch limitation tends to appear in the spectrum as a higher, narrower peak compared to that of a fully developed sea, indicating spectral concentration towards a smaller range of frequencies that is commonly seen in fetch limited regions [8]. To suit the local parameters of other regions, the JONSWAP spectrum can be modified through the alteration of the gamma factor, γ, also known as the spectral peak factor. The JONSWAP formula is given by [9]:

$$S(\omega) = \alpha g^2 \omega^{-5} e^{\left[-1.25\left(\frac{\omega}{\omega_0}\right)^{-4}\right]} \gamma^{e^{\left[\frac{-(\omega-\omega_0)^2}{2\tau^2\omega_0^2}\right]}} \quad (4)$$

The JONSWAP spectrum is a two-parameter spectrum as a function of ω_0 and γ, where ω_0 is the dominant frequency of the sea spectra and γ is the peak factor governed by the significant wave height, H_s, and the peak period, T_p, of the modeled sea state [9]. The zeroth factor, α, and the shape parameter, τ, are empirically stable enough to be considered constants. In ideal situations, wave data originating from a particular region of concern are preferred so that spectral analysis can be performed for site-specific peak factor values. For a sea spectrum that represents North Sea conditions, the peak factor value is set at 3.3. Fig. 3 shows the relationship of the peak factor on the spectral shape, indicating fetch effects [9].

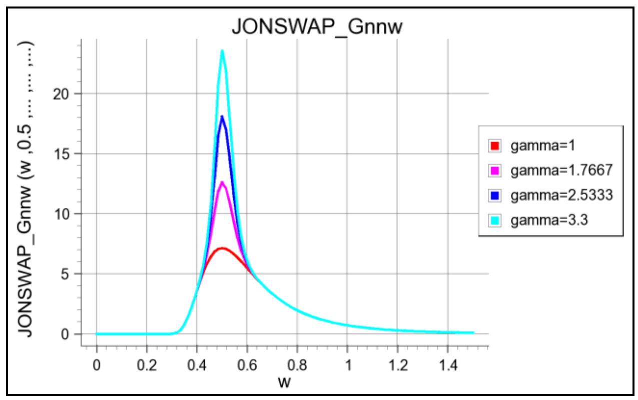

Fig. 3. Effect of peak factor on spectral shape

III. METHODOLOGY

A. Experimental Setup

A truss spar model was developed to a scale of 1:100. The Froude scaling factors are applied based on laws of similitude. Figs. 4 and 5 show the dimensions of the truss spar model.

The truss spar model is placed in the wave tank and three wave probes were placed to measure the wave height generated in the tank. The experimental layout is illustrated in Fig. 6. The model is then subjected to unidirectional random waves generated using the JONSWAP spectrum, for which the wave modeling parameters of the sea states have been scaled down using the laws of similitude. The modeling parameters are shown in Table 1.

The generated wave height and the structural responses were measured and recorded. The time histories are then transformed into their frequency counterparts to obtain the power spectral density functions and the FRF.

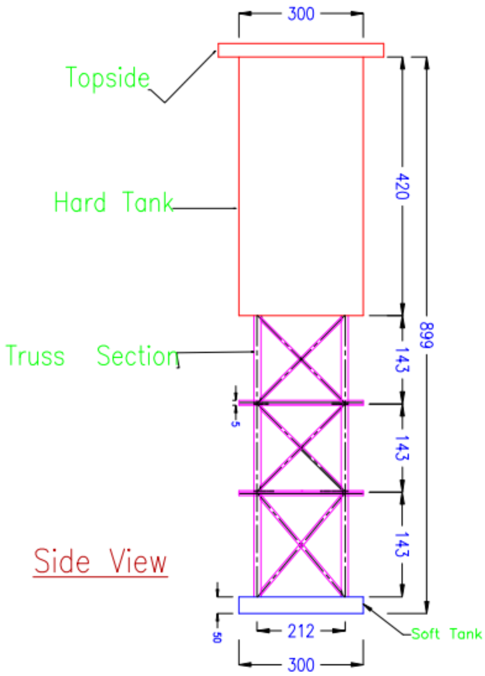

Fig. 4. Dimensions of the truss spar (in mm)

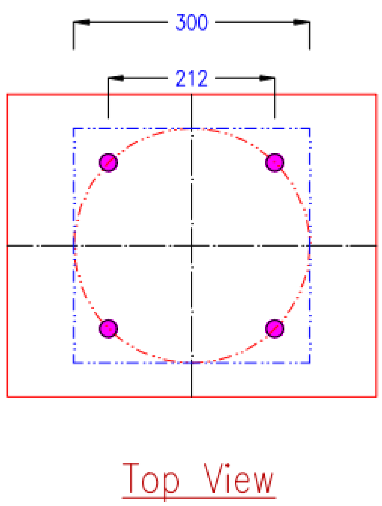

Fig. 5. Dimensions of the truss spar (in mm)

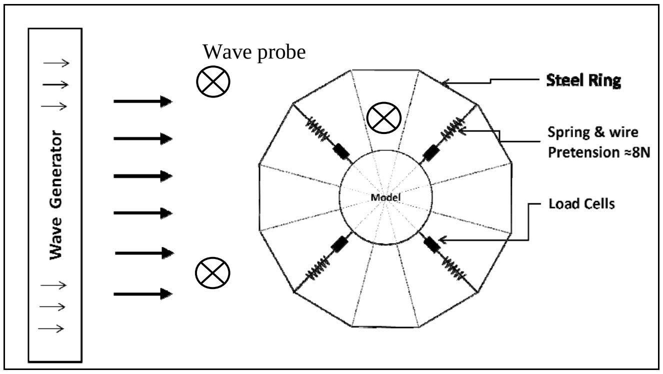

Fig. 6. Experimental layout

TABLE I. MODELED SEA STATE PARAMETERS

Test No.	Scale Wave Frequency [Hz]	Actual Wave Frequency [Hz]	Actual Wave Period [s]	Significant wave height [m]
RD2	0.71	0.071	14.0	0.06
RD3	1.00	0.100	10.0	0.05
RD4	0.83	0.083	12.0	0.07

B. Froude Scaling Factors

Based on laws of similitude, the Froude scaling factors are applied when developing a scaled model of the truss spar as well as modeling sea state parameters such as wave period and significant wave height. While some scales are linearly related, others are related by nonlinear coefficients. For example, mass and distance are linearly related while time is scaled by the square root of the scale factor – in our case, it is the square root of 100. Table 2 lists the scaling factors used to scale down the parameters used in developing the 1:100 truss spar model as well as developing the modeled sea state parameters [10].

TABLE II. MODELED SEA STATE PARAMETERS

Parameter	**Froude Scale Ratio**
Length	100
Wave period	10
Wave height	100

The table implies that for a 50-m length, the scaled down model would be 0.5 m long. To model a 14-s wave period, the scaled down version should have a period of 1.4 s. Similarly, results that are obtained from model tests can be scaled back to full-scale results for realistic representations.

C. Assumptions and Limitations

In modeling the wave parameters, several assumptions and limitations are identified to interpret certain phenomena in the results presented. These modeling limitations manifest themselves due to the physical setup of the wave tank, similitude laws of scaling and the ability of the wave generators to model a random process.

It has to be noted that due to testing in a wave tank, the structural model is susceptible to the effects of side wall currents. Experiments tend to suffer primarily due to the finite width of the wave tanks and the effects of waves on the side boundaries on the hydrodynamic processes. Approaching waves create longshore currents that result in rip currents along the side walls, therefore in the long run it creates large-scale circulation within the wave tank [11].

Due to the scale of the model which is capped at 1:100, it is imperative that the testing parameters are subjected to the laws of similitude. This is to ensure that the parameters are scaled down appropriately to represent full-scale conditions. Even with similitude laws in place, experimental anomalies such as small shifts in mass distribution of model structure and angle of wave direction compared to a full-scale model is very likely to result in amplified errors when scaled back. Furthermore, physical characteristics such as material stiffness cannot be scaled down and therefore must be stated as part of the initial assumptions of this test.

It is important to note that the generation of waves in the wave tank is, at best, wide-sense stationary. This is due to the algorithm employed by the wave generators which uses the sum of Fourier waves based on specified sea spectra to generate the spectral distribution; they are periodic in nature, resulting in stationarity. This is different from actual sea states which are naturally stochastic processes and therefore are usually nonstationary [8].

D. *Modulation Instability of Generated Waves*

It is well documented that the effects of wave deformation and modulation as it propagates is also known as the Benjamin-Feir instability. Research in this area has indicated that unstable wave components are highly related to sideband space, δ and initial wave steepness, ε [17]. As a rule of thumb, wave steepness of less than 0.11 is considered the occurring point of wave modulation instability. However, the recurrence of the initial state of a wave time series is observed which demonstrates that the spreading of frequencies during the propagation of a wave is not adversely affected by the modulation effects. Furthermore, the time invariant nature of Fast Fourier Transform allows sufficiently long sampling rates of the wave basin to reduce the impact of wave modulation instability on the spectral results through averaging effect.

IV. RESULTS AND DISCUSSION

As the tests are carried out based on zero-degree wave heading, the structure is not expected to yield responses in the sway, roll and yaw DOFs. However, an inspection of the results yields that some responses are still obtained in the aforementioned DOFs due to factors such as side-wall effects, efficiency of wave attenuators and issues related to human precision such as perfect orthogonal placement of the structure in the wave tank with respect to the wave heading. Nonetheless, the results for the three DOFs are discarded so as not to affect the interpretation of results for the heave, pitch and surge DOFs.

In the experimental setup, the waves were generated based on the distribution of waves as described by the JONSWAP spectrum empirical formula given by Eq. 4. Fig. 7 shows the power spectral density functions for the three different generated waves: RD2, RD3 and RD4.

The spectra are seen to concur very closely with theoretical experimental setup values where RD2 test obtains 0.65Hz, RD3 at 0.80 Hz and RD4 at 1.10 Hz peak frequencies. There is lower spectral content present around the peak frequencies and this is consistent with the generation of the JONSWAP spectrum wave. Peak frequency magnitudes theoretically should yield lower magnitudes at higher frequencies and this characteristic is present in the experimental results.

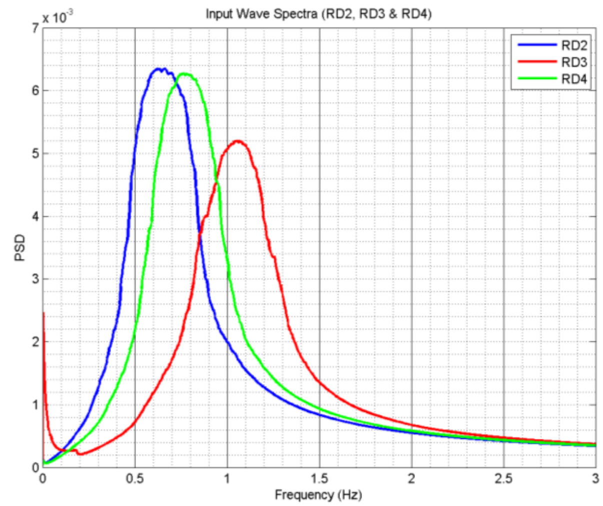

Fig. 7. Power spectral density plots of the varying wave frequencies

A. *Stationarity of Truss Spar Responses*

The method of frequency domain analysis employed in this study utilizes the Fast Fourier Transform which inherently requires the time series to be at least wide-sense stationary [12]. In order to satisfy this criterion, the time series is analyzed for stationarity using the autocorrelation function. The simulation of sea states in the wave tank is done by generating a sequence of Fourier waves in random order in which the finite sum of these waves will result in a spectral variance in the frequency domain which is equivalent to the specified JONSWAP spectrum. Due to the stochastic generation of these waves, obtaining a strictly stationary condition in the time series is not possible. This assumption requires the data to be strictly stationary in the first and second moments. In practical sense, such a time series would not be possible under naturally occurring stochastic processes and therefore an assumption is utilized whereby only the first moments of the time series is required to be stationary with respect to time [13]. A parametric approach to evaluating stationarity of a time series can be derived from literature where if a time series is nonstationary, then the sample autocorrelation function will neither cut off nor die down quickly, but rather will die down extremely slowly [2]. This is performed through qualitative evaluation based on the rate at which the autocorrelation plot dies down compared to the dominant forcing frequencies. Figs. 8 and 9 clearly demonstrate that the wave and truss spar response time series are weakly stationary. There is a periodic oscillation observed in the autocorrelation plots which indicates that there is a dominant wave or structural mode present in the time series.

B. *Response Power Spectral Density Functions*

Due to the geometrical symmetry of the truss spar, the response power spectra will be confined to the heave, pitch and surge spectrums, as shown in Figs. 10, 11 and 12. The power spectra are studied for their response frequency against the varying forcing wave frequencies.

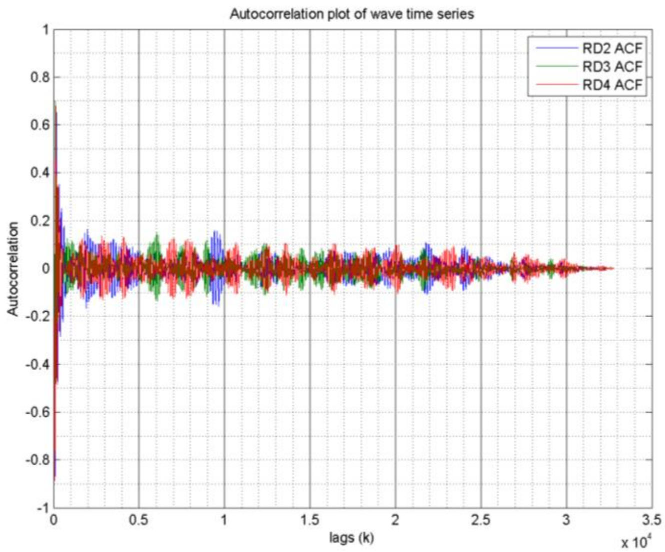

Fig. 8. Autocorrelation plot of the wave time series

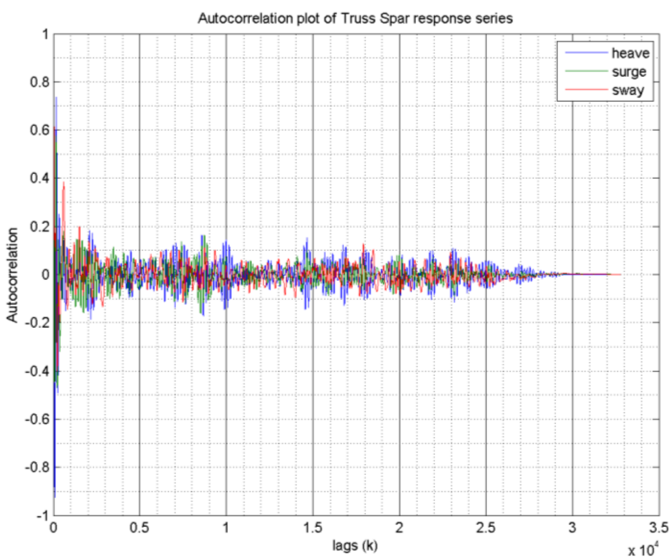

Fig. 9. Autocorrelation plot of truss spar response time series

The response power spectra indicate in general very significant peaks in the first mode of the respective DOFs. Heave model peak frequencies are in the range of 0.6 Hz while surge frequency peaks at 0.18 Hz and pitch frequency peaks at 0.35 Hz. There is a persistent second mode oscillating at the range of 0.6 Hz to 1.0 Hz, which is related to the forcing frequencies (RD2, RD3 and RD4) of the wave components imposing their oscillations upon the structure. The heave peak frequency in this experimental setup is very close to the forcing wave frequency of RD2, which is 0.65 Hz. This would result in very high responses due to near-resonance conditions. However, this did not occur in the experimental setup due to the high heave damping provided by the truss spar heave plates [15]. The dynamic amplification factor usually seen in near-resonance conditions were not present and displayed response magnitudes similar to the first mode of excitation. This observation is elaborated further in the FRF plots.

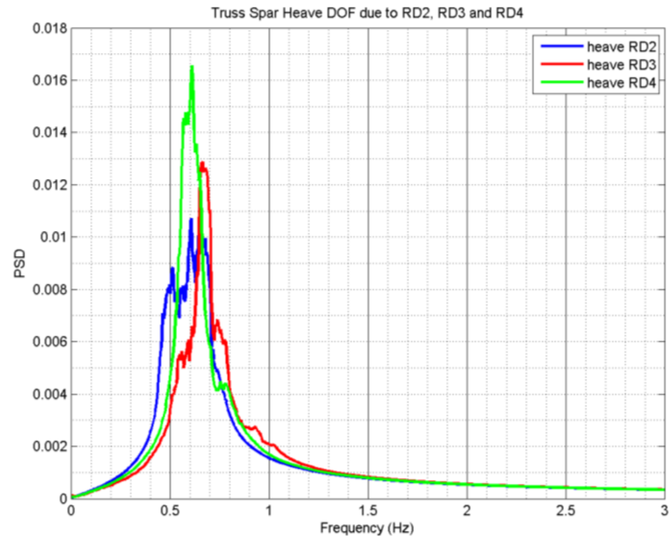

Fig. 10. Power spectral density functions for the heave DOF

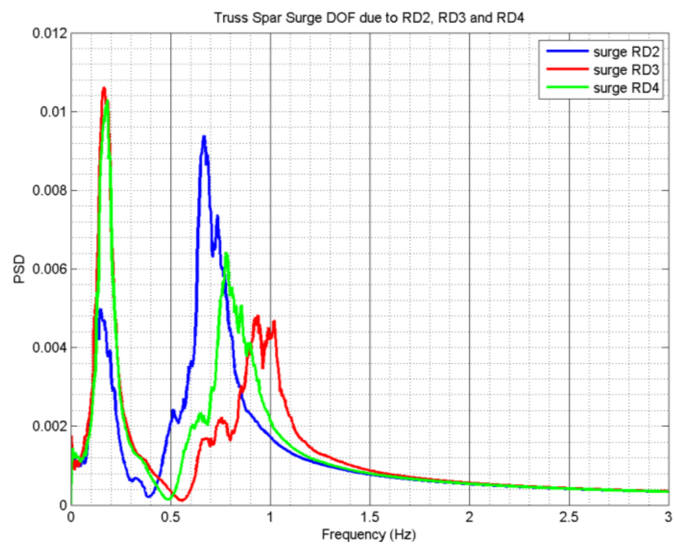

Fig. 11. Power spectral density functions for the surge DOF

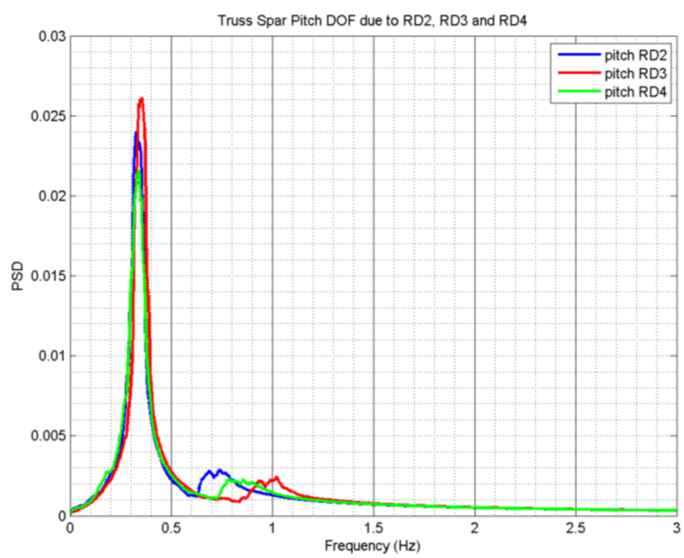

Fig. 12. Power spectral density functions for the pitch DOF

Table 3 summarizes the fundamental frequencies of the truss spar model and their full-scale equivalents.

TABLE III. FUNDAMENTAL FREQUENCIES OF TRUSS SPAR

DOF	Model frequency [Hz]	Equivalent full-scale frequency [Hz]	Equivalent full-scale fundamental period [s]
Heave	0.60	0.060	16.7
Surge	0.18	0.018	55.5
Pitch	0.35	0.035	28.6

C. Frequency Response Function (FRF) Analysis

Interpretations of the response power spectra are akin to performing modal analysis of a structure. It is an output-based only analysis where the spectral peaks cannot be clearly identified until further information is verified and fed into the system. Frequency response functions (FRF) enable the identification of responses that is characteristic to a structure, thus separating spectral peaks belonging to the forcing frequency from those corresponding to the natural frequencies of the structure. The FRFs can be referred to as the identity of the structure under varying excitations. Figs. 12, 13 and 14 illustrate the FRFs for heave, surge and pitch motions.

The truss spar design utilized in this experiment is rather unique as the heave natural frequency has near-resonance values with the forcing wave frequencies. This creates a near-resonance conditions which in this particular setup did not manifest itself due to the high damping provided by the heave plates in the vertical direction. This can be observed from the FRF plots where the heave FRF magnitudes are far lower than those of surge and pitch DOFs. This indicates that the truss spar design has excellent resistance to motion in the heave direction even under near-resonance conditions. As the fundamental frequency of the heave DOF is closely spaced to the dominant frequency of the waves, the FRF plots tend to have a broader spectrum band.

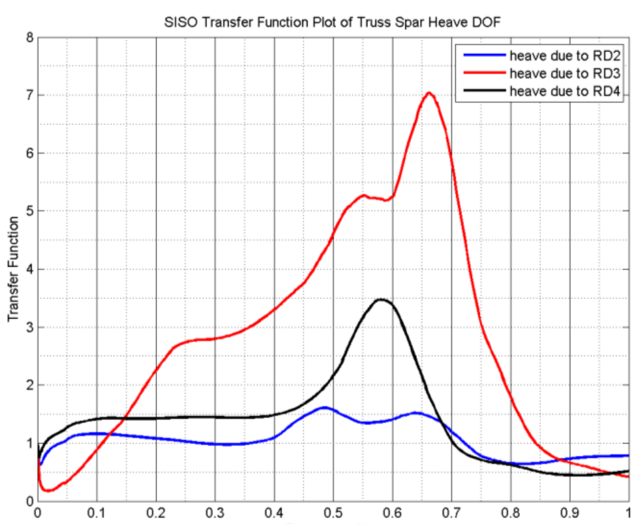

Fig. 12. Frequency response function for the heave DOF

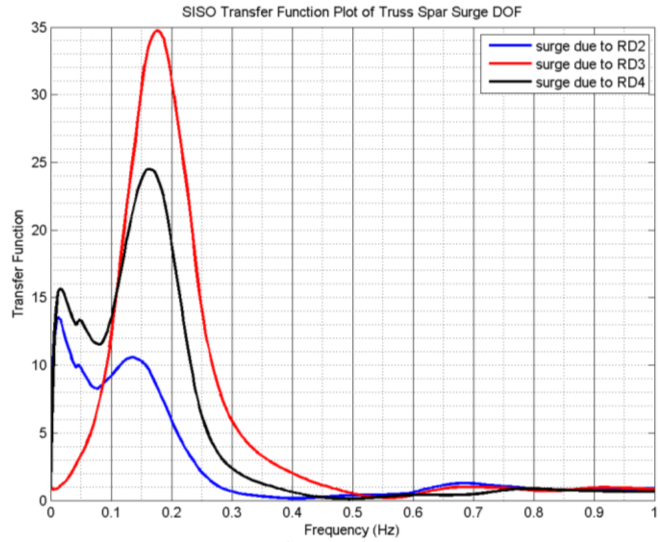

Fig. 13. Frequency response function for the surge DOF

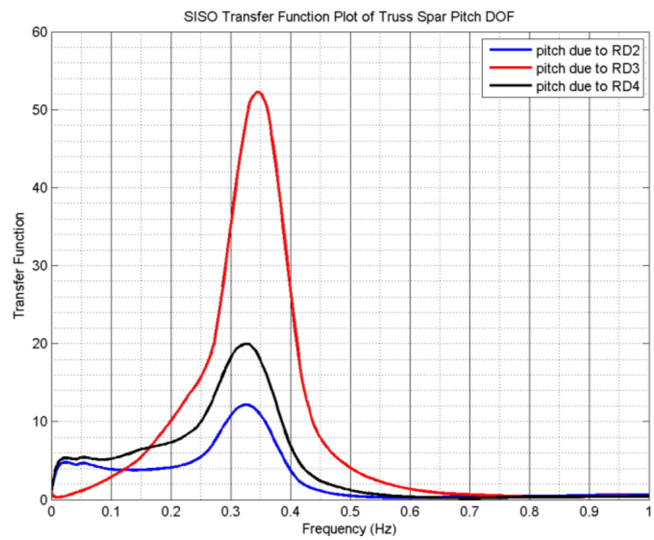

Fig. 14. Frequency response function for the pitch DOF

From the FRF plots, the FRF analysis effectively highlights the peaks that are related to the response of the structure; the peak related to the forcing wave frequencies are eliminated leaving behind the peaks related to the system identity in the fundamental mode. The natural frequencies of the surge and pitch DOFs are far below the forcing wave frequencies. The attenuation of motions in the surge and pitch direction are eliminated to near zero magnitudes from the 0.5 Hz to 1.0 Hz range. However there is significant response in the low lying frequencies of the FRF. These are related to second order waves that significantly impact the spar structure especially in the surge direction. These second order waves are a result of closely spaced wave frequencies interacting with each other, producing waves with very long periods [16]. Motion control with respect to these waves is highly dependent on the lateral stiffness provided by the mooring lines and drag on the structure.

V. CONCLUSION

This paper has shown that the response of the truss spar during a head sea condition is only significant in the heave, surge and pitch DOFs. The heave plates utilized in the design of the truss spar has tremendously reduced heave motions as seen in the FRF plots. This is due to the additional damping and added mass provided by the added surface of the heave plates. The forcing wave frequency was simulated close to the heave natural frequency and despite such conditions the truss spar has exhibited very low responses in the heave direction. This characteristic will be essential to extreme sea states whereby heave-inducing forces can be significant. This will enable the protection of more sensitive components such as risers. Effect of second order waves are also seen in the surge FRFs. These low frequency waves are a result of interaction between closely space wave frequencies and impact DOFs with long natural periods, thus reducing motion resistance of the truss spar especially in the surge direction.

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