Experimental Investigation of Hydrodynamic Force Coefficients on Rigid Vertical Cylinders

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***Abstract*— The present study is alleged to create appreciation about the importance of experimental estimation of the hydrodynamic force coefficients for making use in the structural design aspects of offshore structures. Model tests were conducted in the wave tank, on cylinders of four different diameters with wave characteristics pertaining to the major offshore sites of Malaysia. This particular paper, however, is limited to a discussion on rigid, vertical cylinders in regular waves. The theoretical wave forces were estimated using Morison Equation. From the results presented, it is found that both the drag and inertia coefficients are showing an increasing trend with an increase in the cylinder diameters of the members considered, like the variation of the total wave forces. Variation of the force coefficients in relation with the wave heights and time periods pertaining to the locations studied, is random in nature. The experimentally determined values of hydrodynamic force coefficients are considered to give modifications in the design and thus on the dimensional parameters of the platforms, which is deemed to enhance the economic efficiency of the offshore projects.**

Keywords — hydrodynamic force coefficients; regular waves; cylinder diameter; wave force; wave height; time period

I. INTRODUCTION

The economy of construction and assembly of many offshore structures like jacket platforms, rely on the precise estimation of wave forces on small diameter slender tubular structures. To accomplish a good level of accuracy of the estimated wave forces, the hydrodynamic force coefficients which are to be used in the theoretical wave force estimations should be accurate. The dependence of the drag and inertia coefficients in wavy and harmonic flows on the Reynolds number, Keulegan-Carpenter number, and the relative roughness ratio has been clearly established by Keulegan and Carpenter [1] and Chakrabarti [2]. The outcome of the above works depicts the varying nature of these force coefficients. Owing to this varying nature of the coefficients the conventional design of offshore structures are more conservative rather than optimum. The experimental determination of these force coefficients is thus significant.

The cylindrical structures find numerous applications in the offshore edifice in comparison with the square and rectangular sections, which could easily be fabricated. This is attributed to the phenomena of narrow wake development and low vortex shedding around the circular cylindrical bodies resulting in a lower wave force on the structure [3]. Wave force estimation on small diameter cylindrical structures is done by making use of the well-known Morison equation [4]. In the Morison regime, the forces depend on two empirical coefficients, the drag and inertia coefficients. The values of these coefficients vary with both wave and model parameters. Extensive appraisal of hydrodynamics around single cylinders is studied by Williamson [5]. Methods of estimation of hydrodynamic force coefficients in regular waves are given by Chakrabarti [2] and that in random waves is presented by Isaacson *et al.* [6]. Recent works on wave forces on two circular cylinders in tandem arrangement, by Kurian *et al.*[7] showed the significant modification on the wave forces in comparison to that acting on a singly existing cylinder in the waves. The wave characteristics and the hydrodynamic force coefficients to be used in the theoretical wave force calculation were obtained from Petronas Technical Standards [8].

The results presented in this paper itself suggest that experimental determination, rather than the theoretical estimation, of wave forces is inevitable for the accurate and economic design of the structural members. In the present study, regular wave characteristics pertaining to four Malaysian offshore locations are considered and the variation of hydrodynamic force coefficients acting on cylindrical members of four different diameters are investigated. The conventional values of these drag and inertia coefficients, adopted from the design codes are expected to give a design towards the very conservative side. The large budget offshore projects could be made economically more efficient by an optimum and safe design rather than a very conservative design.

A. Theoretical Wave Force Calculation

The in-line wave forces on small diameter cylinders are estimated theoretically using the well-known Morison equation. The total force is composed of drag as well as inertia part as given in (1). The influence of various wave and prototype parameters on the drag and inertia coefficients are significant and plays an important role in the estimation of accurate wave forces acting on the structure.

$$F_T = 1/2\ \rho\ D\ u\ |u|\ C_d + \pi/4\ \rho\ D^2\ \alpha\ C_m \qquad (1)$$

Where

F_T = Total wave force
ρ = Density of sea water
D = Cylinder diameter
u = Horizontal water particle velocity
α = Horizontal water particle acceleration
C_d = Drag coefficient
C_m = Inertia coefficient

$$u = \frac{\pi\ H \cosh(ks)}{T \sinh(kd)} \cos\theta \qquad (2)$$

$$\alpha = \frac{2\ \pi{*}\pi\ H \cosh(ks)}{T{*}T \sinh(kd)} \sin\theta \qquad (3)$$

Where

H = Maximum wave height
T = Time period
k = Wave number
s = Distance from sea bed to wave surface
d = Water depth
θ = Phase angle

B. Modelling criteria and Scaling

Knowing that, in the case of water flow with a free surface, the gravitational effects predominate, the Froude's scaling law was employed for the scaling of parameters [9]. The maximum water depth obtainable in the UTP wave tank is 1 m, so in the test runs pertaining to each of the locations a scale factor of 1:λ is used, where λ is the water depth corresponding to that particular location.

The common variables found in the study of wave mechanics are identified and using Froude's law and the scale factor λ, the suitable multipliers to be used to obtain the prototype parameters from model data are shown in Table I.

TABLE I. SCALE FACTOR MULTIPLIERS

Variable	**Unit**	**Scale factor**
Wave Height	L	λ
Wave Period	T	$\sqrt{\lambda}$
Wave Force	$ML\ T^{-2}$	λ^3
Wave Length	L	λ
Particle Velocity	$L\ T^{-1}$	$\sqrt{\lambda}$
Particle Acceleration	$L\ T^{-2}$	1

II. EQUIPMENTS AND METHODOLOGY

The experimental determination of wave force coefficients are made by conducting wave tank tests for different site-specific wave characteristics.

A. Wave Force Sensors

Since the wave forces acting on the model test cylinders are of less magnitude, very sensitive and accurate force sensors were designed and fabricated exclusively for the present study. The design details of the force sensor were published in an earlier paper [10]. The material properties of the fabricated force sensors are given in Table II.

TABLE II. MATERIAL PROPERTIES OF FABRICATED FORCE SENSOR

Material Used	Aluminum-6061
Material property	**Value**
Mass Density (kg/m^3)	2710
Yield Strength (MPa)	275
Ultimate Tensile Strength (MPa)	310
Young's Modulus (GPa)	68.9
Poisson's Ratio	0.33
Shear Modulus (GPa)	25.9

Fabricated wave force sensor of dimensions 0.10 m * 0.05 m * 0.03 m is shown in Fig. 1. One face of the force sensor is provided with four strain gauges, which again is connected to suitable data loggers for the recording of wave forces.

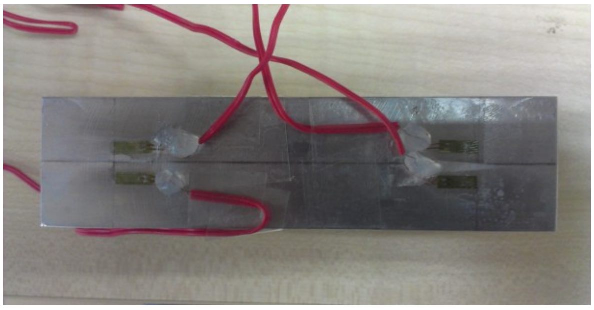

Fig. 1. Wave force sensor

B. Wave Tank Tests

Model tests were performed in the wave tank of Universiti Teknologi PETRONAS, Perak, Malaysia, which is 22 m long, 10 m wide and 1.5 m deep. The wave tank facility of UTP is shown in Fig. 2.

Fig. 2. Wave tank facilities - Wave generator

The wave generator consists of sixteen individual paddles that could generate waves propagating in different directions. Other components that are integrated in the wave tank tests are the wave probes, strain gauges and the specially fabricated force sensors. Unidirectional, regular waves with maximum model wave height and maximum model time period of 0.14 m and 1.33 s respectively are studied.

The completion of experimental set-up, includes model instrumentation as well as the wave tank instrumentation. This entails procedures like setting all the necessary model connections for force sensors, data loggers and force data monitoring unit. Also the wave tank is instrumented with wave probes to record the wave profiles. Calibration of force sensors and wave probes are also rendered.

Wave characteristics and water depth pertaining to Peninsular Malaysia Operation (L1), Baram Delta (L2), Samarang (L3) and Erb West (L4) are considered for the present study and are given in Table III.

TABLE III. MODEL & PROTOTYPE WAVE CHARACTERISTICS

Location	Model Wave Height (m)	Prototype wave height (m)	Model Time period (S)	Prototype Time period (S)	Water Depth (m)
L1	0.121	8.44	1.0	8.34	70
L2	0.087	6.5	1.03	8.90	75
L3	0.138	6.9	1.33	9.40	50
L4	0.108	6.7	1.0	7.90	62

For recording the forces acting on the cylinders, the cylinders are installed in the wave tank as a cantilever beam fixed at the top, as shown in Fig. 3.

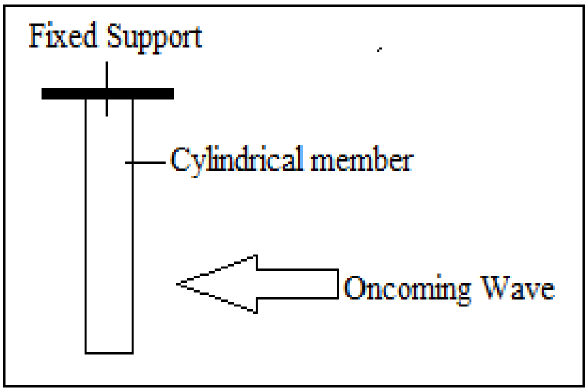

Fig. 3. Sketch of installed cylinder in wave tank

The test specimens are cylindrical pipes of wet length 0.95 m, made from galvanized iron, with specifications as given in Table III. Since the scale factors used in the four locations are different, the prototype cylinder diameters corresponding to each location also is different and is given in Table IV.

TABLE IV. CYLINDER SPECIFICATIONS

Location	Model Diameter (m)	Scale factor	Prototype Diameter (m)
L1	0.022	70	1.54
	0.027		1.89
	0.034		2.38
	0.042		2.94
L2	0.022	75	1.65
	0.027		2.03
	0.034		2.55
	0.042		3.15
L3	0.022	50	1.10
	0.027		1.35
	0.034		1.70
	0.042		2.10
L4	0.022	62	1.36
	0.027		1.67
	0.034		2.11
	0.042		2.60

C. *Data Analysis*

The wave data is recorded at a frequency of 100 readings per second and the force data is sampled at a frequency of 200 readings per second. The actually recorded wave heights were comparable with the input parameters in the wave generator. In-order to eliminate any possible errors which could be caused due to the use of wave heights which is given in Table III, the actually generated maximum wave heights calculated from each test run, and was made use in the theoretical wave force calculation.

The forces recorded during the model time range 30 s until 90 s only were considered for the estimation of hydrodynamic force coefficients. This was done to avoid any interruption in the force and wave profile recordings by the reflected waves from the idle end of the wave tank. From the recorded model force and model wave profile time series, the corresponding prototype values were obtained by using suitable scale factors.

Morison equation is given as:

$$F_T = 1/2\, \rho\, D\, u\, |u|\, C_d + \pi/4\, \rho\, D^2\, \alpha\, C_m$$

$$= A\, C_d + B\, C_m$$

In the Morison equation, since the values of u and α are time dependent, as given in (2) and (3), the theoretically estimated wave forces vary at each time instant, so are the values of A and B. Multiple linear regression technique employing the least square methods is made use for the computation of this unknown drag and inertia coefficients. The forces measured at different time instants and the coefficients A and B are used as the input data for regression analysis. The time step considered for the estimation of wave forces and the coefficients A and B is 0.1 seconds. Thus the 60 s range under study will be yielding 600 equations, with only C_d and C_m as the unknowns.

The method adopted for the evaluation of the force coefficients is depicted in the sequence as shown in Fig. 4.

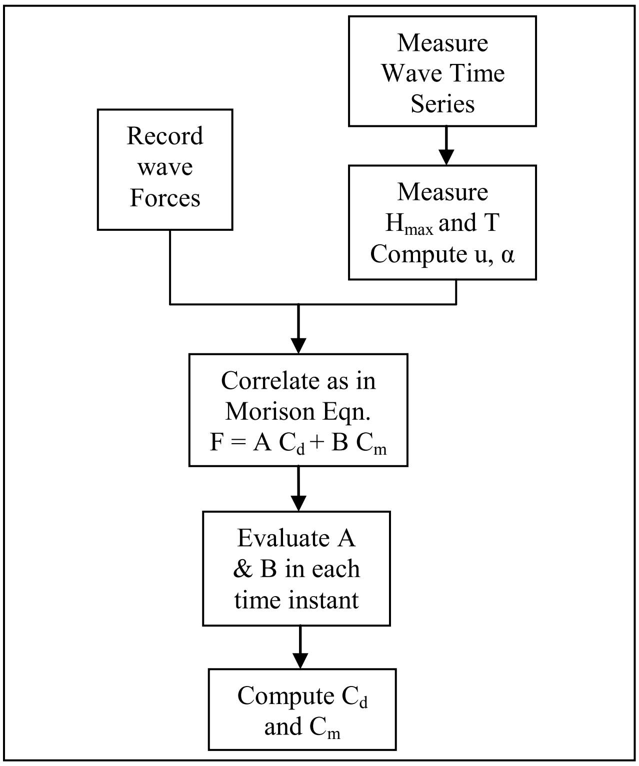

Fig. 4. Flowchart of the steps in hydrodynamic force coefficient computation

Validations of the results are done by comparison of the results with those in the existing literature and also by making use of relevant structural analysis software packages. The results of which are to be presented in another paper.

III. Results and Discussion

In this section, bending moment independence of the specially fabricated force sensors is presented at the outset. A comparison of the experimental and theoretical wave forces are furnished thereafter. The variations of force coefficients with cylinder diameters are endowed thereafter. Also, the influence of the wave parameters at different locations considered, on the wave force coefficients are studied and presented.

A. Response of the Fabricated Force Sensor

Specially fabricated force sensors are used for the wave force measurements. The significant feature of this force sensor is that the in-line wave forces independent of the bending moments is accurately recorded. To verify the independence of the bending moments on the wave forces recorded, different concentrated loads were applied at different lever arms. This load is applied perpendicular to the face of the strain gauges. Fig. 5 shows the response of wave force sensor for different loads at different lever arms.

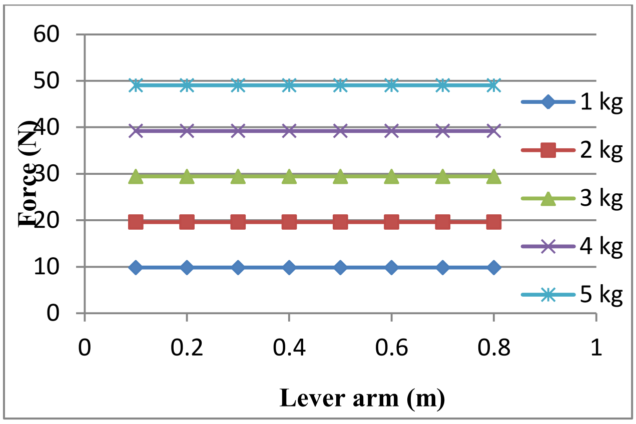

Fig. 5. Response of fabricated force sensor

The constant nature of the recorded forces, irrespective of the lever arm, as shown in Fig. 4, depicts the independence of the recorded forces on the bending moments, when the forces are acting perpendicular to the face of the strain gauges. In the actual wave tank tests also the forces are acting perpendicular to the face of the strain gauges. To further simplify, only the horizontal in-line wave forces are recorded by the force sensor.

B. Experimental and Theoretical Wave Force Comparison

At all the four locations considered, a comparison of the theoretical and experimental forces acting on the studied cylinder diameters is made. The results are presented graphically in Fig. 6. The comparison shows that the wave forces are related directly with the cylinder diameters. It also clearly represents the overestimation of the wave forces due to the adoption of conventional theoretical method, suggesting the significance of site-specific experimental determination of the wave forces.

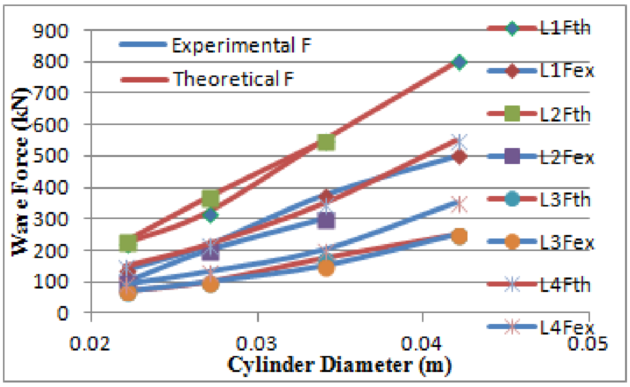

Fig. 6. Theoretical & experimental wave force comparison

C. Variation of Force Coefficients with Cylinder Diameter

All the four cylinder diameters were tested in wave characteristics pertaining to all the four locations considered for the study. The data from the wave tank tests are used for the regression analysis. The force coefficients are estimated after each test run. The drag and inertia coefficients for the cylinder diameters, averaged over the different locations are given in Table V.

TABLE V. DRAG AND INERTIA COEFFICIENTS

Cylinder Diameter (m)	Drag Coefficient	Inertia Coefficient
0.022	0.588	1.035
0.027	0.670	1.066
0.034	0.749	1.115
0.042	0.890	1.362

The variation of estimated hydrodynamic force coefficients with the cylinder diameters is presented in Fig. 7. It shows that both drag and inertia coefficients vary in direct proportion with the cylinder diameters. The trend in the variation of force coefficients with the cylinder diameters is very similar to the variation of wave forces with the same, as given in Fig. 6.

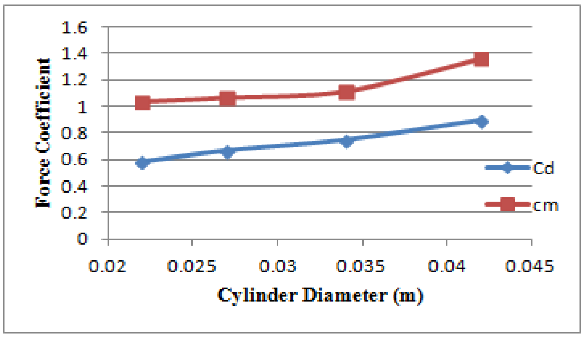

Fig. 7. Variation of force coefficients with cylinder diameter

The cylinders were tested in the waves individually. There was no neighboring members covering these members from the waves, thus these tested cylinders comply with the definition of isolated members, as per [8]. Fig. 8 shows the scatter of drag and inertia coefficients in the studied cases.

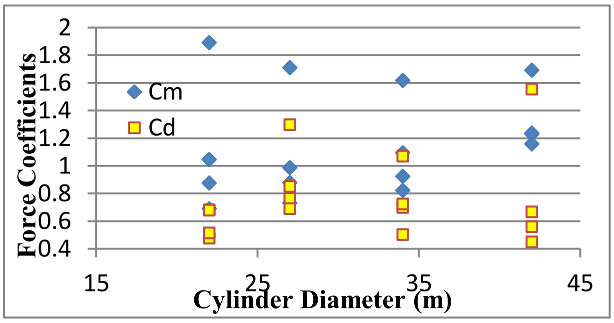

Fig. 8. Variation of force coefficients with wave heights

Clause 4.5.b of [8] demands the use of constant values 1.0 and 2.0 respectively for drag and inertia coefficients, in the design of isolated members. From the results presented in Table V and Fig. 8, it is clearly comprehended that both the force coefficients are lesser than these constant values, 1.0 and 2.0. On an average, the drag coefficient is over predicted by 38% and the inertia coefficient is over predicted by 75%. This high exaggeration of the coefficient values will lead to the overestimation of the wave forces on the cylindrical members and thus leads to uneconomical design of offshore structures.

D. Variation of force coefficients with wave parameters at different locations

An effort to generalize the values of the hydrodynamic force coefficients at the studied locations also is made in the present paper. An attempt is made to figure out if there is any particular trend in the variation of force coefficients when the wave heights and time periods are increased. The drag and inertia coefficient values obtained for different diameters at the same location is averaged together and a single value is obtained. The value of coefficients thus obtained is considered as an average value at that particular location.

The variation of force coefficients with the maximum wave heights, corresponding to each location is given in Fig. 9. The locations were considered in the order of increasing wave heights.

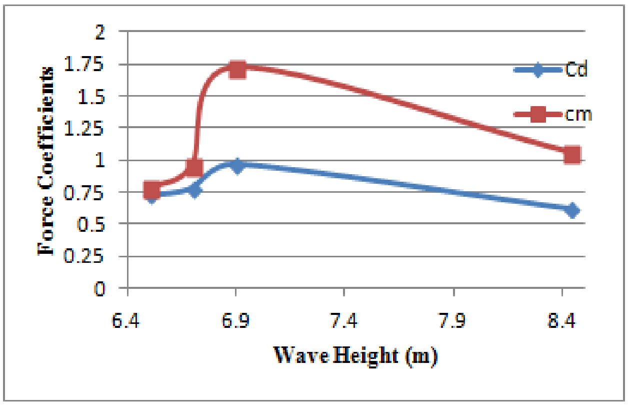

Fig. 9. Variation of force coefficients with wave heights

It is comprehended from this plot that there is no specific increasing or decreasing trend for the force coefficients when the wave heights are increased. Similar is the case with time period also, considering the different locations in the order of increasing time periods, as given in Fig. 10.

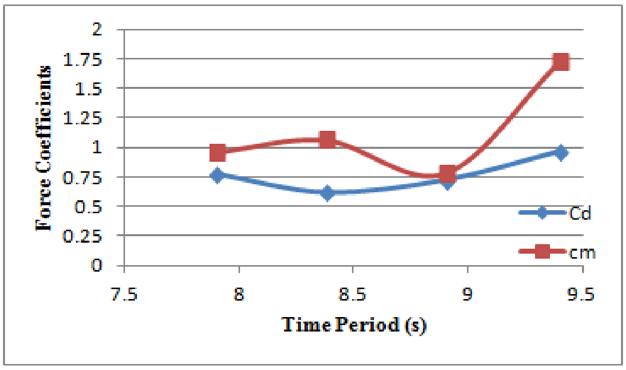

Fig. 10. Variation of force coefficients with time period

The plots in Fig. 9 and Fig. 10 depict the variation of coefficients with wave heights and time periods separately. In real scenario these coefficients at a location is influenced simultaneously by these two wave parameters. A combined plot of wave heights, time periods and force coefficients pertaining to each location is given in Fig. 11, which again is showing no particular trend in the variation of the coefficients at the locations considered.

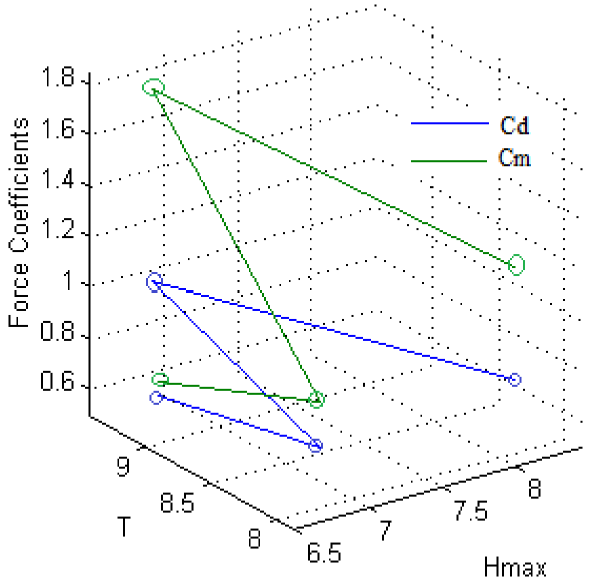

Fig. 11. Force coefficients Vs Time period Vs Wave height

IV. Conclusions

The study reported in this paper is a part of an investigation for the determination of site-specific hydrodynamic force coefficients, to be used in the Morison equation, for Malaysian offshore locations. The major conclusions emerged from the present study are:

- Theoretical calculation overestimates the force values on the cylinders in comparison with the experimental wave forces at the locations considered.
- The values of hydrodynamic force coefficients increase when the diameters of the cylindrical members are increased. Most importantly, the varying nature of the force coefficients is established, contravening the adoption of constant values for these coefficients by the design standards.
- For the studied cases of cylinder diameters and locations the hydrodynamic force coefficients are fairly over-predicted which may lead to an economically unfavorable design of the platform members.
- In the studied locations, the variation of force coefficients with wave heights and time periods are random in nature.

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