Representing EEG Source Localization Using Finite Element Method

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***Abstract*— Finite Element Method (FEM) is a numerical tool usually used to solve various problems related to electromagnetic field, biomechanics, stress analysis etc. In this paper, the finite element is proposed as a solution to the localization problem of the active sources inside the brain. This localization is termed as the EEG Inverse problem. The solution to EEG inverse problem with less localization error, high resolution and less computational complexity leads to better understanding of human brain behavior and helps neurologist and neurosurgeons in curing various neurological disorders. The implementation of the FEM in solving EEG inverse problem is explained and then a pseudo code in MATLAB is designed and explained for the application to solve the problem. However, for illustration purpose, the solution to the 1D electromagnetic problem through FEM is plotted to elaborate graphically the procedure.**

Index Terms— ***EEG Inverse Problem, Finite Element Method, Global Matrix, Poisson Equation.***

I. Introduction

Finite element method is a very useful numerical tool which is used for solution of boundary value problems which are defined by a differential equation with a set of boundary conditions [1], [2], [3], [4]. FEM is utilized for the solution of various field problems which include Stress analysis, Biomechanics, Heat transfer, Fluid flow, Biomedical and Electromagnetics etc. In FEM analysis, the domain under observation is discretized into small sub domains (finite elements) and the unknown quantity inside the element is interpolated based on the value provided at nodes [5]. Hence FEM goes through seven major steps which are: Mesh generation, Selection of proper interpolation/basis function, conversion of integral equations into linear equations i.e. weak formulation, Assemblage of the system, imposing of boundary conditions (Dirichlet or Neumann), Solution of linear equations by using common linear algebra techniques and finally post processing of the results. These processes for solution of EEG inverse problem are discussed at detailed in this paper.

Along with other field problem, FEM is also used for head modeling which is used an initial step for forward problem solution. After that, source localization of brain signals is carried out which are responsible for neuronal activity inside the brain [6]. This localization of the sources which is called EEG inverse problem is helpful to diagnose pathological and physiological disorders related to neuro science surgery and research [7]. These neurological disorders include epilepsy and brain tumors. The other neuroimaging techniques such as MRI, fMRI etc. are also used for the analysis of brain behavior but here EEG is discussed because of its low cost and ease of resource availability [8-12]. Source localization estimates the location and orientation of dipole which minimize the least-squared error between the calculated and measured potential. The localization is carried out by solving a series of forward problems for potential until the error between exact and estimated is minimized [13]. Therefore, the localization or inverse problem can be treated as series of forward solutions. In this paper, mathematical modeling of FEM used for formulation of forward solution is discussed which provides a base for solution of ill -posed EEG inverse problem.

II. APPLIED FEM FOR EEG INVERSE PROBLEM

For the understanding of FEM applied to solution, let's have a look into forward problem first. The forward problem can be defined as; given the position, orientation and magnitudes of current dipoles with known geometry and electrical conductivity of head, one has to calculate the electric potential over the scalp.

This forward problem can be elaborated mathematically by following Poisson equation [14]:

$$\nabla.(\sigma\nabla V) = \sum I_s(r) \qquad (1)$$

With Neumann boundary condition having first partial derivative of electric potential (V) equals to zero. In the above equation, σ is conductivity, V is potential created due to current sources in the brain and I_s is volume current density.

For the ideal situations, the current dipole can be modeled as having two point sources of opposite polarity having large current density I_0 separated by small distance *d*. mathematically it can be described as:

$$I_s = \lim_{d\to 0} I_0\left[\delta\left(r - r_s - \frac{d}{2}\right) - \delta\left(r - r_s + \frac{d}{2}\right)\right] \quad (2)$$

Here *d* is serration distance such that $d.I_0 = P$, the dipole strength [13]. The solution of FEM can be achieved with two different approaches namely, Direct method and subtraction method. Here direct method with related mathematical modeling with assumed parameters is discussed at length.

A. Direct Method FEM Approach

The direct method is one of the approaches used to solve governing equation (1) for EEG inverse problem. Assume that mesh consisting of N nodes, then the potential can be expressed as:

$$V(r) = \sum_{n=1}^{N} V_n h_n \quad (3)$$

Here h_n is basis function related to a particular node *n*. The FEM procedure starts form meshing of the system under observation. So the conducting region is meshed into triangular elements to calculate *V(r)* by solving following system of equation:

$$[U_{mn}][V_n] = [J_n^d] \quad (4)$$

Here $[U_{mn}]$ is a $N \times N$ matrix and J_n^d is N dimensional vector whose nth component can be calculated as:

$$J_n^d = -Id\nabla h_n.P \quad (5)$$

Now let's evaluate and explain the forward formulation for a triangular element by solving 2D Poisson equation. In this method, total potential for EEG measurements are first calcula--ted and then FEM equations are generated by direct impleme--ntation of dipole source into the model assuming source and sink separated by small distance.

B. Mathematical modelling

As the first step for FEM, the solution domain Ω is discretized into smaller elements. The physical quantity i.e. electrical potential (V) in this case, to be determined is estimated at points called nodes. The potential values for the system are determined by the solution of (5). For the, estimation of U_{mn} , one has to solve following matrix:

$$\begin{bmatrix} \iiint \sigma\nabla f_1.\nabla f_2 dV & \iiint \sigma\nabla f_1.\nabla f_2 dV & \ldots\iiint \sigma\nabla f_1.\nabla f_N dV \\ \iiint \sigma\nabla f_2.\nabla f_1 dV & \iiint \sigma\nabla f_2.\nabla f_2 dV & \ldots\iiint \sigma\nabla f_2.\nabla f_N dV \\ \vdots & \ddots & \ldots \\ \iiint \sigma\nabla f_N.\nabla f_1 dV & \iiint \sigma\nabla f_N.\nabla f_2 dV & \ldots\iiint \sigma\nabla f_N.\nabla f_N dV \end{bmatrix} \quad (6)$$

This is so called global matrix which needs to be calculated for proposed inverse problem. Each entry of U_{mn} is filled according to the contribution of element in the solution domain. This can be expressed by another square matrix u_{ij}. The dimension of this matrix is equal to the number of nodes related to the element. The entries of u_{ij} can be written as:

$$u_{ij} = \sigma_e \iiint \nabla f_i^e.\nabla f_j^e dV \quad (7)$$

Here σ_e is the conductivity of the element and f_i^e is the basis function for the element.

For the triangular elements, the shape function in terms of local co-ordinates (r, s) as [15]:

$$h_1(r,s) = 1 - r - s \quad (8)$$

$$h_2(r,s) = r \quad (9)$$

$$h_3(r,s) = s \quad (10)$$

The x and y co-ordinates for any physical quantity inside the element in terms of element functions can be given by:

$$\begin{aligned} x &= x_1 f_1^e + x_2 f_2^e + x_3 f_3^e \\ x &= x_1 + (x_2 - x_1)r + (x_3 - x_1)s \end{aligned} \quad (11)$$

$$\begin{aligned} y &= y_1 f_1^e + y_2 f_2^e + y_3 f_3^e \\ y &= y_1 + (y_2 - y_1)r + (y_3 - y_1)s \end{aligned} \quad (12)$$

Here $(x_1, y_1),(x_2, y_2),(x_3, y_3)$ are co-ordinates of three nodes for triangular element. The gradients for the element function used for solution of forward problem for triangular elements

$$\nabla f_i^e = \frac{\partial f_i^e}{\partial x} x^\wedge + \frac{\partial f_i^e}{\partial y} y^\wedge \quad (13)$$

are calculated by solving following equations in chain rule differentiation:

$$\frac{\partial f_i^e}{\partial r} = \frac{\partial f_i^e}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f_i^e}{\partial y}\frac{\partial y}{\partial r} \quad (14)$$

And $$\frac{\partial f_i^e}{\partial s} = \frac{\partial f_i^e}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f_i^e}{\partial y}\frac{\partial y}{\partial s} \quad (15)$$

These relations can be expressed in matrix form as:

$$\begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial s} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \end{bmatrix}\begin{bmatrix} \frac{\partial f_i^e}{\partial x} \\ \frac{\partial f_i^e}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_i^e}{\partial r} \\ \frac{\partial f_i^e}{\partial s} \end{bmatrix} \quad (16)$$

Hence the gradient by using above mathematical manipulation can be written as:

$$\nabla f_1^e = \frac{1}{2A_e}\left[(y_2 - y_3)\hat{x} + (x_3 - x_2)\hat{y}\right] \quad (17)$$

$$\nabla f_2^e = \frac{1}{2A_e}\left[(y_3 - y_1)\hat{x} + (x_1 - x_3)\hat{y}\right] \quad (18)$$

$$\nabla f_3^e = \frac{1}{2A_e}\left[(y_1 - y_2)\hat{x} + (x_2 - x_1)\hat{y}\right] \quad (19)$$

In final, all these mathematical relations can help to calculate entries of u_{ij}. Hence, uij can be defined as:

$$[u_{ij}] = (\sigma_e)(A_e)\begin{bmatrix} \nabla f_1^e.\nabla f_1^e & \nabla f_1^e.\nabla f_2^e & \nabla f_1^e.\nabla f_3^e \\ \nabla f_2^e.\nabla f_1^e & \nabla f_2^e.\nabla f_2^e & \nabla f_2^e.\nabla f_3^e \\ \nabla f_3^e.\nabla f_1^e & \nabla f_3^e.\nabla f_2^e & \nabla f_3^e.\nabla f_3^e \end{bmatrix} \quad (20)$$

In this way, the entries for global matrix U_{mn} are calculated. This computation will help us to put the values in (5) for the calculation of unknown quantity inside the element i.e. electrical potential for this EEG inverse problem. Note that the vector on right hand side is assumed to be unity for ease of calculations. However, its value can be altered to have different forward solutions. The error can be calculated between estimated and exact value by various conventional methods and equations. Hence the steps needed for implementation of FEM for EEG Inverse solution (with the basic assumption of isotropic medium and modeling source as dipole) can be outlined as:

a) The conducting region (brain) is meshed with triangular elements with n number of nodes.

b) The potential value at every node is calculated by using Eq. 4 given above.

c) The Neumann boundary condition is applied for unique solution.

d) The system of equation is produced with non-singularity property after imposition of choosing of reference node from the grid with fixed potential value.

e) Solution of system equations is sorted out by using any inversion technique.

f) The accuracy of the designed algorithm can be checked by using error relation (Eq. 21). This Eq. shows the difference between exact solution and analytical solution obtained by FEM.

C. Pseudo Code in MATLAB

After having an in depth sight into the calculation procedure, now let's have a look into implementation of system via computer program. Explained below is a pseudo program for implementation of the system.

- Define input variables involve such as co-ordinate value $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, conductivity for elements (σ_e), number of elements (Ne) and Number of nodes (No) etc.
- Define parameters to calculate u_{ij}.
- Initialize Global matrix and right hand side vector as:

```
G=zeros(No);
J=zeros(No,1);
```

- Run the loop through the elements for element connectivity matrix.

```
for ele=1:Ne
    ec(i,1)=i;
    ec(i,2)=i+1;
end
```

Note: ec means element connectivity.

- Double loop through local node of each element

```
for i=1:2
  for j=1:2
      G(ec(ele,i),ec(ele,j))=
   G(ec(ele,i),ec(ele,j))+
   u(i,j);
  end
end
```

- Apply Boundary condition if any is imposed.
- Desired unknown variable is calculated by any standardized linear algebra technique (gauss Jordan, Gauss elimination) by inverting the Global matrix (U_{mn} in this case).
- The results so obtained are plotted against the exact solution.
- Finally the error is observed between the exact solution and estimated solution.

III. RESULTS AND DISCUSSION

The strategy discussed above for the implementation of FEM for solution of EEG inverse problem can be used by following the steps described. The crux lies in finding out the co-ordinate values for building up of the Global matrix. The co-ordinate values can be evaluated by using mesh generation software for 2D and 3D problem. However, the inverting procedure is done with Gauss-Jordan or other helpful linear algebra technique. The difference between measured electrical potential and the estimated potential can be found by following formula:

$$Error = \frac{\sum_{i=1}^{N}\left|V^{exact} - V^{FEM}\right|}{\sum_{i=1}^{N}\left|V^{exact}\right|} \quad (21)$$

This error can give some quantitative measure between the measured and estimated value. By following same strategy for FEM solution, below are the results produced for 1D electromagnetic problem. These results are produced [5] to illustrate the comparison between exact and numerical solution calculated by finite element method modeling for a simple 1D electromagnetic problem as shown in Fig. 1 and Fig. 02. The Idea can be extended for any physical model such as head modeling for the forward solution of EEG inverse problem as discussed earlier by taking into consideration system constraints for Inverse problem related to brain source localization.

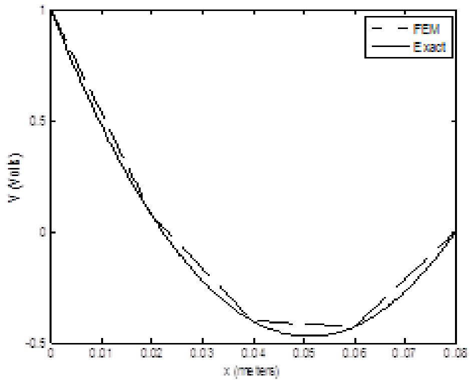

Fig.1. Plot for comparison between Exact and FEM Solution

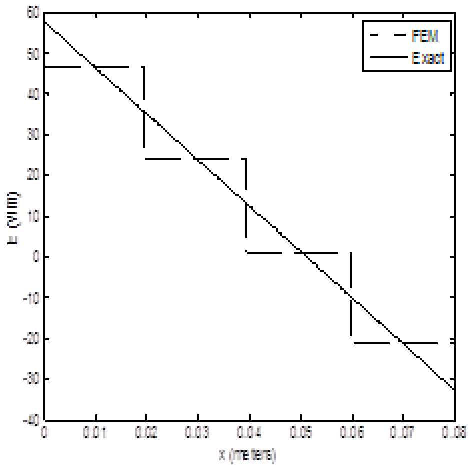

Fig.2. Plot for comparison between Exact and FEM Solution

IV. Conclusion

Finite element method (FEM) is useful numerical tool for solving of various field problems. Here, mathematical modeling along with the pseudo code for the numerical solution of EEG inverse problem using FEM is produced. The implementation can be carried out by using toolboxes such as MATLAB for the solution of EEG inverse problem with less localization error, less computational time and more resolution. The theoretical outline is produced in modeling section for the implementation of FEM as head modeling tool for EEG Inverse problem. However, the sample graphs for 1D EM problems are produced for illustrating working of error function for it. The results so obtained are to be compared with other localization techniques for the sketching of best solution for this ill-posed problem.

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