Applied Mechanics and Materials Vols. 465-466 (2014) pp 149-154
© (2014) Trans Tech Publications, Switzerland
doi:10.4028/www.scientific.net/AMM.465-466.149

Unsteady Magnetohydrodynamic Free Convection Flow of a Radiative Fluid past an Infinite Vertical Plate with Constant Heat and Mass Flux

Narahari Marneni[1, a], Sowmya Tippa[2, b] and Rajashekhar Pendyala[3, c]

[1, 2]Fundamental and Applied Sciences Department, Universiti Teknologi PETRONAS, 31750 Tronoh, Malaysia

[3]Chemical Engineering Department, Universiti Teknologi PETRONAS, 31750 Tronoh, Malaysia

[a]marneni@petronas.com.my, [b]t.sowmya88@gmail.com, [c]rajashekhar_p@petronas.com.my

Keywords: Free convection, thermal radiation, magnetohydrodynamics, heat flux, mass flux, vertical plate.

Abstract. Theoretical analysis of unsteady magnetohydrodynamic free convection flow of a viscous incompressible radiative fluid past an infinite vertical plate with constant heat and mass flux is presented. The dimensionless governing linear partial differential equations have been solved using the Laplace transform technique. The exact solutions for the velocity, temperature and concentration fields are derived. The effects of radiation, magnetic field and buoyancy ratio parameters on the velocity and temperature fields are discussed through graphs. It is found that the velocity increases with increasing radiation parameter whereas it decreases with increasing magnetic field parameter for buoyancy assisted flows.

Introduction

Combined heat and mass transfer plays an important role in many engineering and environmental applications such as the design of chemical processing equipment, cooling towers in power plants, design of space vehicles, distribution of temperature and moisture over agricultural fields and groves of fruit trees, formation and dispersion of fog etc. The radiation effect can be quite significant for some industrial applications such as furnace design, glass production, plasma physics and aircraft re-entry aerothermodynamics that operate at high temperatures. The application of magnetic field also plays an important role in MHD pumps, MHD power generators and the cooling of reactors. The unsteady free convection flow of a viscous incompressible electrically conducting fluid past an accelerated infinite vertical plate with constant heat flux was studied by Chandran et al. [1]. An exact solution of unsteady free convection flow past an impulsively started infinite vertical plate with uniform heat and mass flux was presented by Muthucumaraswamy et al. [2]. Chemical reaction effects on unsteady natural convection flow past an infinite vertical plate with uniform heat flux and variable mass diffusion were investigated by Muthucumaraswamy and Kulandaivel [3]. A numerical solution of the unsteady hydromagnetic free convection flow past an impulsively started vertical plate with uniform heat and mass flux in the presence of thermal radiation was presented by Ramachandra Prasad et al. [4]. Recently, Narahari and Debnath [5] investigated the unsteady magnetohydrodynamic free convection flow of an electrically conducting fluid past an accelerated infinite vertical plate with constant heat flux in the presence of heat generation or absorption. However, the exact solution of unsteady radiative magnetohydrodynamic free convection flow of an electrically conducting fluid past an infinite vertical plate with constant heat and mass flux has not been addressed in the literature.

In the present paper, it is proposed to investigate the unsteady magnetohydrodynamic free convection flow of a viscous incompressible electrically conducting fluid past an infinite vertical plate with constant heat and mass flux in the presence of thermal radiation. Closed form analytical solutions for the velocity, temperature and concentration fields are obtained using the Laplace transform technique. These solutions allow convenient analysis of the physical problem and an understanding of the system parameters on the flow.

Mathematical Analysis

Consider the unsteady free convection flow of an electrically conducting viscous incompressible fluid past an infinite vertical plate. The x' - axis is taken along the plate in the upward direction and the y' - axis perpendicular to the plate into the fluid by choosing an arbitrary point on this plate as the origin. Initially, the plate and the fluid are at the same temperature T'_∞ and concentration C'_∞. At time $t' > 0$, the temperature and concentration levels at the plate are raised at a constant rate. A uniform magnetic field of strength B_0 is applied in the y' direction. The magnetic Reynolds number of the flow is assumed to be small so that the induced magnetic field is neglected in comparison with the applied magnetic field (B_0). All the physical properties of the fluid are assumed to be constant except the density variations with temperature in the body force term. As the plate is of infinite extent in x' direction, all the physical quantities are functions of the space coordinate y' and time t' only and therefore the inertia terms are negligible. It is also assumed that the radiation heat flux in the x' direction is negligible as compared to that in the y' direction. Then, under the usual Boussinesq approximation and neglecting the heat due to viscous dissipation, the free convection flow along the vertical plate can be shown to be governed by the following equations:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) - \frac{\sigma B_0^2}{\rho} u' \tag{1}$$

$$\rho C_P \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

The corresponding initial and boundary conditions are

$$\left.\begin{array}{lllll} t' \le 0: & u' = 0,\ T' = T'_\infty,\ C' = C'_\infty & \text{for all} & y' \ge 0, \\ t' > 0: & \left\{ \begin{array}{l} u' = 0,\ \dfrac{\partial T'}{\partial y'} = -\dfrac{q_w}{k},\ \dfrac{\partial C'}{\partial y'} = -\dfrac{j_w}{D} \\ u' \to 0,\ T' \to T'_\infty,\ C' \to C'_\infty \end{array}\right. & \begin{array}{l} \text{at} \\ \text{as} \end{array} & \begin{array}{l} y' = 0, \\ y' \to \infty \end{array} \end{array}\right\} \tag{4}$$

For an optimally thin constant property gas, the radiative heat flux q_r satisfies the following non-linear differential equation [6]:

$$\frac{\partial q_r}{\partial y'} = 4\alpha\sigma^* (T'^4 - T'^4_\infty) \tag{5}$$

It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature T' using the Taylor series expansion about T'_∞ and neglecting the higher order terms, gives

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{6}$$

In view of (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - 16\alpha\sigma^* T'^3_\infty (T' - T'_\infty) \tag{7}$$

Upon introducing the characteristic length $L = \nu^{2/3} / g^{1/3}$, the non-dimensional quantities are defined as follows:

$$\left.\begin{aligned} &y=\frac{y'}{L}, t=\frac{t'(\nu g)^{1/3}}{L}, u=\frac{u'}{Gr(\nu g)^{1/3}}, \theta=\frac{T'-T'_\infty}{T^*}, C=\frac{C'-C'_\infty}{C^*}, M=\frac{\sigma B_0^2 L}{\rho(\nu g)^{1/3}}=\frac{\sigma B_0^2 L^2}{\mu}, \\ &Gr=\frac{\beta q_w L}{k}=\beta T^*, Gm=\frac{\beta^* j_w L}{D}=\beta^* C^*, N=\frac{Gm}{Gr}, \mathrm{Pr}=\frac{\mu C_P}{k}, R=\frac{16\sigma^*\alpha T'^3_\infty L^2}{k}, \mathrm{Sc}=\frac{\nu}{D}. \end{aligned}\right\} \quad (8)$$

where $T^*=\dfrac{q_w L}{k}$ and $C^*=\dfrac{j_w L}{D}$ are characteristic temperature and concentration, respectively, u', T' and C' are velocity, temperature and species concentration of the fluid near the plate, respectively, T'_∞ and C'_∞ are temperature and species concentration of the fluid far away from the plate, respectively, t' is time, ν is kinematic viscosity, g is acceleration due to gravity, β is volumetric coefficient of thermal expansion, β^* volumetric coefficient of concentration expansion, ρ is density, σ is electrical conductivity, B_0 is applied magnetic field, C_P specific heat at constant pressure, k is thermal conductivity, q_w is heat flux per unit area at the plate and j_w is mass flux per unit area at the plate, D is mass diffusivity, Gr and Gm are thermal and mass Grashof numbers, respectively, μ is coefficient of viscosity, α is radiation absorption coefficient, σ^* is Stefan-Boltzmann constant, Pr is Prandtl number, R is dimensionless radiation parameter, Sc is Schmidt number, N is buoyancy ratio parameter, M is magnetic field parameter (square of the Hartmann number), y is dimensionless coordinate axis normal to the plate, t is dimensionless time, u is dimensionless velocity, θ is dimensionless temperature, and C dimensionless concentration of the species.

In view of equations (8), equations (1), (7) and (3), respectively, take the following non-dimensional forms:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+\theta+NC-Mu \quad (9)$$

$$\mathrm{Pr}\frac{\partial \theta}{\partial t}=\frac{\partial^2 \theta}{\partial y^2}-R\theta \quad (10)$$

$$\mathrm{Sc}\frac{\partial C}{\partial t}=\frac{\partial^2 C}{\partial y^2} \quad (11)$$

and the corresponding non-dimensional initial and boundary conditions are

$$\left.\begin{aligned} &t\le 0:\ u=0,\ \theta=0,\ C=0 \qquad \text{for all} \quad y\ge 0, \\ &t>0:\begin{cases} u=0,\ \dfrac{\partial\theta}{\partial y}=-1,\ \dfrac{\partial C}{\partial y}=-1 & \text{at} \quad y=0, \\ u\to 0,\ \theta\to 0,\ C\to 0 & \text{as} \quad y\to\infty. \end{cases} \end{aligned}\right\} \quad (12)$$

The equations (9), (10) and (11) subject to the initial and boundary conditions (12) are exactly solved by the usual Laplace transform technique without any restriction and the solutions are derived for different cases with the help of inverse Laplace transform formulas given in [5].

$$C(y,t)=\frac{1}{\sqrt{\mathrm{Sc}}}F_1(y\sqrt{\mathrm{Sc}},t) \quad (13)$$

$$\theta(y,t)=\frac{1}{\sqrt{\mathrm{Pr}}}F_2(y\sqrt{\mathrm{Pr}},a_1,0,t) \quad (14)$$

Case I: $\mathrm{Pr}\ne 1,\ \mathrm{Sc}\ne 1$

$$u(y,t) = a_5F_4(y,M,t) + a_4F_3(y,M,-a_2,a_1,t) - a_4F_3(y,M,0,a_1,t) - a_5F_3(y,M,a_3,0,t) \\ - a_4F_2(y\sqrt{\Pr},a_1,-a_2,t) + a_4F_2(y\sqrt{\Pr},a_1,0,t) + a_5F_2(y\sqrt{\mathrm{Sc}},0,a_3,t) - a_5F_1(y\sqrt{\mathrm{Sc}},t) \tag{15}$$

Case II: $\Pr \neq 1,\ \mathrm{Sc} = 1$

$$u(y,t) = -a_6\, F_4(y,M,t) + a_4F_3(y,M,-a_2,a_1,t) - a_4F_3(y,M,0,a_1,t) - a_4F_2(y\sqrt{\Pr},a_1,-a_2,t) \\ + a_4F_2(y\sqrt{\Pr},a_1,0,t) + a_6\, F_1(y,t) \tag{16}$$

Case III: $\Pr = 1,\ \mathrm{Sc} \neq 1$

$$u(y,t) = a_5F_4(y,M,t) - a_5F_3(y,M,a_3,0,t) + a_7F_3(y,M,0,R,t) + a_5F_2(y\sqrt{\mathrm{Sc}},0,a_3,t) \\ - a_7F_2(y,-R,0,t) - a_5F_1(y\sqrt{\mathrm{Sc}},t) \tag{17}$$

where,

$$a_1 = \frac{R}{\Pr},\ a_2 = \frac{\Pr a_1 - M}{\Pr - 1},\ a_3 = \frac{M}{(\mathrm{Sc}-1)},\ a_4 = \frac{1}{\sqrt{\Pr}(1-\Pr)a_2},\ a_5 = \frac{N}{a_3\sqrt{\mathrm{Sc}}(1-\mathrm{Sc})},$$

$$a_6 = \frac{N}{M}, a_7 = \frac{1}{(R-M)},$$

$$F_1(z_1,t) = L^{-1}\left\{\frac{\exp\left(-z_1\sqrt{s}\right)}{s^{3/2}}\right\} = 2\sqrt{t/\pi}\exp\left(-\frac{z_1^{\,2}}{4t}\right) - z_1\,\mathrm{erfc}\left(\frac{z_1}{2\sqrt{t}}\right),$$

$$F_2(z_1,z_2,z_3,t) = L^{-1}\left\{\frac{\exp\left(-z_1\sqrt{s+z_2}\right)}{(s-z_3)\sqrt{s+z_2}}\right\}$$

$$= \frac{\exp(z_3t)}{2\sqrt{z_2+z_3}}\left[\exp(-z_1\sqrt{z_2+z_3})\,\mathrm{erfc}\left\{\frac{z_1}{2\sqrt{t}} - \sqrt{(z_2+z_3)t}\right\}\right. \\ \left. - \exp(z_1\sqrt{z_2+z_3})\,\mathrm{erfc}\left\{\frac{z_1}{2\sqrt{t}} + \sqrt{(z_2+z_3)t}\right\}\right],$$

$$F_3(z_1,z_2,z_3,z_4,t) = L^{-1}\left\{\frac{\exp\left(-z_1\sqrt{s+z_2}\right)}{(s-z_3)\sqrt{s+z_4}}\right\}$$

$$= \frac{\sqrt{z_2+z_3}\exp(z_3t)}{2(z_3+z_4)}\left[\exp(-z_1\sqrt{z_2+z_3})\,\mathrm{erfc}\left\{\frac{z_1}{2\sqrt{t}} - \sqrt{(z_2+z_3)t}\right\}\right. \\ \left. - \exp(z_1\sqrt{z_2+z_3})\,\mathrm{erfc}\left\{\frac{z_1}{2\sqrt{t}} + \sqrt{(z_2+z_3)t}\right\}\right] \\ + \frac{i\sqrt{z_4-z_2}\exp(-z_4t)}{2(z_3+z_4)}\left[\exp(iz_1\sqrt{z_4-z_2})\,\mathrm{erfc}\left\{\frac{z_1}{2\sqrt{t}} + i\sqrt{(z_4-z_2)t}\right\}\right. \\ \left. - \exp(-iz_1\sqrt{z_4-z_2})\,\mathrm{erfc}\left\{\frac{z_1}{2\sqrt{t}} - i\sqrt{(z_4-z_2)t}\right\}\right],$$

$$F_4(z_1, z_2, t) = L^{-1}\left\{\frac{\exp\left(-z_1\sqrt{s+z_2}\right)}{s^{3/2}}\right\}$$

$$= \sqrt{t/\pi}\exp\left(-\frac{z_1^{\,2}}{4t} - z_2 t\right) - \left(\frac{2tz_2 + z_1\sqrt{z_2} + 1}{4\sqrt{z_2}}\right)\exp(z_1\sqrt{z_2})\operatorname{erfc}\left(\frac{z_1}{2\sqrt{t}} + \sqrt{z_2 t}\right)$$

$$+ \left(\frac{2tz_2 - z_1\sqrt{z_2} + 1}{4\sqrt{z_2}}\right)\exp(-z_1\sqrt{z_2})\operatorname{erfc}\left(\frac{z_1}{2\sqrt{t}} - \sqrt{z_2 t}\right),$$

F_1, F_2, F_3 and F_4 are dummy functions, and z_1, z_2, z_3 and z_4 are dummy variables.

Results and Discussion

In order to gain a physical insight into the problem, the velocity and temperature profiles have been drawn for different values of radiation, magnetic field and buoyancy ratio parameters in Figs. 1 to 4. Note that the thermal and concentration buoyancy forces act in the same direction when $N > 0$ (aiding buoyancy force) and they oppose each other when $N < 0$ (opposing buoyancy force). The case $N = 0$ corresponds to the situation when there is no buoyancy force effect from mass diffusion.

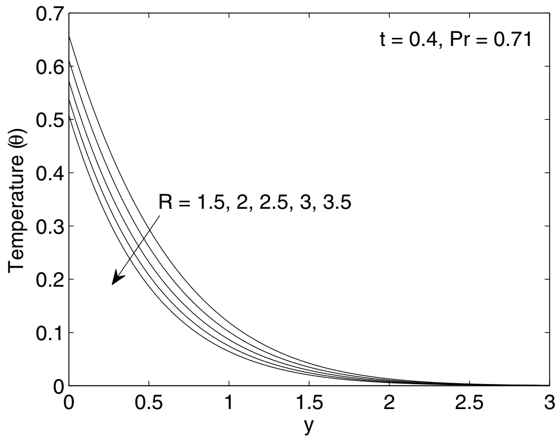

Fig. 1 Temperature profiles for different *R*

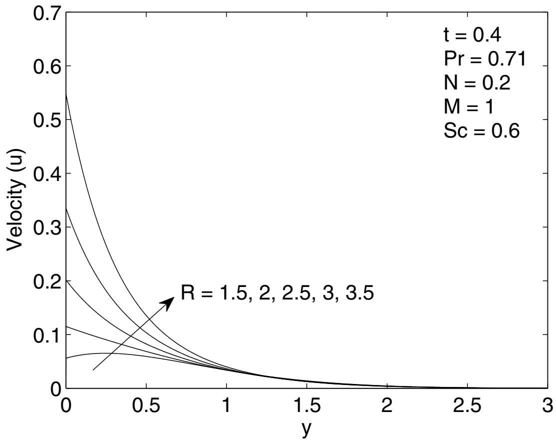

Fig. 2 Velocity profiles for different *R*

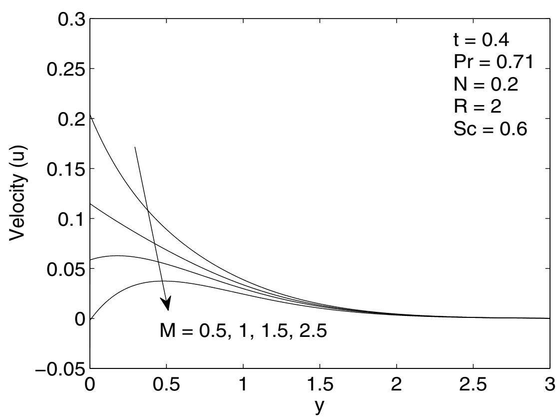

Fig. 3 Velocity profiles for different *M*

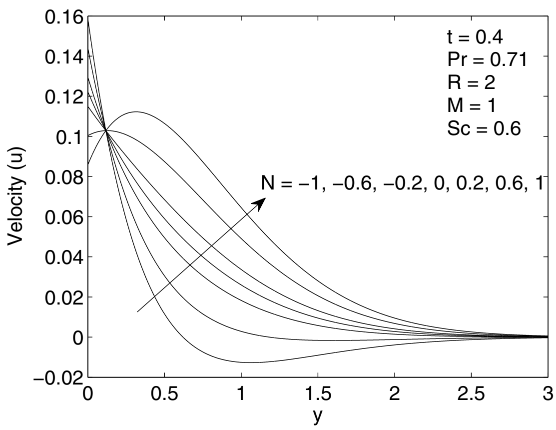

Fig. 4 Velocity profiles for different *N*

In Fig.1, the temperature profiles for different values of thermal radiation parameter are depicted. It is clear from Fig. 1 that the fluid temperature decreases with increasing radiation parameter values. The velocity profiles for different values of radiation parameter are shown in Fig. 2. From Fig. 2 it is observed that the velocity increases with increasing radiation parameter in the presence of aiding buoyancy force $(N > 0)$. The influence of magnetic field parameter on the velocity field is shown in Fig. 3. From this figure it is observed that the velocity decreases with increasing magnetic field parameter due to the resistive type force called the Lorentz force which has tendency to slow down the fluid motion in the boundary layer flow. The influence of buoyancy ratio parameter on the velocity field is shown in Fig. 4. From Fig. 4 it can be seen that the fluid velocity increases with increasing aiding buoyancy force and the fluid velocity decreases with increasing opposing buoyancy force in the boundary layer. However, an opposite behavior in the fluid velocity is encountered adjacent to the plate.

Conclusions

An exact solution to the problem of unsteady magnetohydrodynamic free convection flow past an infinite vertical plate with constant heat and mass flux in the presence of thermal radiation is obtained with the help of Laplace transform technique. Closed form solutions for the velocity, temperature and concentration fields are presented. It is found that the fluid velocity increases with increasing radiation parameter and aiding buoyancy force whereas it decreases with increasing magnetic field parameter and opposing buoyancy force.

Acknowledgements

The authors greatly acknowledge the financial support from the Ministry of Higher Education (MOHE), Malaysia under the Fundamental Research Grant Scheme (FRGS/1/2012/TKO1/ UTP/02/06).

References

[1] P. Chandran, N. C. Sacheti, A. K. Singh, Unsteady hydromagnetic free convection flow with heat flux and accelerated boundary motion, J. Phys. Soc. Jpn. 67 (1998) 124-129.

[2] R. Muthucumaraswamy, P. Ganesan, V. M. Soundalgekar, Heat and mass transfer effects on flow past an impulsively started vertical plate, Acta Mech. 146 (2001) 1-8.

[3] R. Muthucumaraswamy, T. Kulandaivel, Chemical reaction effects on moving infinite plate with uniform heat flux and variable mass diffusion, Forschung im Ingenieurwesen 68 (2003) 101-104.

[4] V. Ramachandra Prasad, N. Bhaskar Reddy, R. Muthucumaraswamy, Transient radiative hydromagnetic free convection flow past an impulsively started vertical plate with uniform heat and mass flux, Theoret. Appl. Mech. 33 (2006) 31-63.

[5] M. Narahari, L. Debnath, Unsteady magnetohydrodynamic free convection flow past an accelerated vertical plate with constant heat flux and heat generation or absorption, ZAMM. Z. Angew. Math. Mech. 93 (2013) 38-49.

[6] M. Narahari, Effects of thermal radiation and mass diffusion on free convection flow past an impulsively started infinite vertical plate with ramped temperature, Acta Technica 57 (2012) 435-450.