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# Newtonian Heating and Mass Transfer Effects on Free Convection Flow past an Accelerated Vertical Plate in the Presence of Thermal Radiation

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**Abstract.** The unsteady free convection flow past an infinite vertical plate with Newtonian heating has been studied in the presence of thermal radiation for a uniformly accelerated plate and an exponentially accelerated plate. The problem is solved under the conditions of (i) uniform wall concentration (UWC) and (ii) uniform mass flux (UMF) by Laplace transform technique. Closed form analytical expressions for the velocity and the skin-friction are obtained for various cases. It is observed that an increase in the buoyancy ratio parameter leads to a decrease in the skin-friction.

**Keywords:** Newtonian heating, natural convection, unsteady flow, thermal radiation, heat transfer, mass transfer, accelerated vertical plate.

**PACS:** 44.40.+*a*, 44.20.+*b*, 44.05.+*e*, 44.25.+*f*, 44.27.+*g* 

#### INTRODUCTION

The unsteady free convection flow of an incompressible viscous fluid past an impulsively started infinite vertical plate resulting from the combined buoyancy forces of heat and mass transfer was first investigated analytically by Soundalgekar<sup>1</sup>. Later, a number of studies were performed on the unsteady free convection flow along a moving infinite vertical plate due to heat and mass transfer for different thermal and concentration boundary conditions<sup>2-14</sup>. Merkin<sup>15</sup> considered a different driving mechanism for the natural convection flow set up by Newtonian heating from the bounding surface i.e. the heat transfer from the surface was taken to be proportional to the local surface temperature. In subsequent studies, Lesnic et al.<sup>16-18</sup> and Pop et al.<sup>19</sup> investigated the effect of Newtonian heating on free convection flow adjacent to a vertical or horizontal plate as well as a slightly inclined plate embedded in a porous medium. Chaudhary and Jain<sup>20</sup> investigated the unsteady free convection flow past an impulsively started infinite vertical plate with Newtonian heating using Laplace transform technique. Mebine and Adigio<sup>21</sup> analyzed the unsteady free convection flow with radiative heat transfer along an infinite vertical porous plate with Newtonian heating. Salleh et al.<sup>22</sup> presented a numerical solution for the twodimensional boundary layer flow and heat transfer of a viscous and incompressible fluid over a stretching sheet with Newtonian heating. Narahari and Ishak<sup>23</sup> presented an exact solution of the unsteady free convection flow past an accelerated infinite vertical plate with Newtonian heating in the presence of thermal radiation. Narahari and Nayan<sup>24</sup> performed an analytical study of the free convection flow along an impulsively started infinite vertical plate with Newtonian heating in the presence of thermal radiation and constant mass diffusion. Recently, Narahari and Dutta<sup>25</sup> investigated the effects of thermal radiation and mass transfer on unsteady free convection flow of an optically dense viscous incompressible fluid along an infinite vertical plate with Newtonian heating. However, the unsteady free convection flow past an accelerated infinite vertical plate with Newtonian heating under the effects of thermal radiation and mass diffusion has not been studied in the literature.

The aim of the present study is to investigate the effect of thermal radiation on unsteady free convection flow past an accelerated infinite vertical plate with Newtonian heating in case of (i) uniform wall concentration (UWC) and (ii) uniform mass flux (UMF) boundary conditions. Exact solutions of the governing partial differential equations have been derived with the Laplace transform technique. This study will be useful in chemical, aerospace and other engineering applications.

#### **MATHEMATICAL ANALYSIS**

and heat transfer of a id over a stretching Narahari and Ishak<sup>23</sup> Consider the flow of a viscous incompressible fluid past an infinite vertical plate with Newtonian heating in the presence of thermal radiation and mass diffusion. The x' - axis is along the plate in the International Conference on Fundamental and Applied Sciences 2012

vertically upward direction and the y' - axis is taken normal to the plate. Initially, the plate and the adjacent fluid are at the same temperature  $T'_{\infty}$  and concentration  $C'_{\infty}$  in a stationary condition. At time t' > 0, the plate begins to move in its plane with a velocity of  $U_0 f(t')$ , where  $U_0$  is a constant and f(t') is a function of time t'. The concentration level at the plate is raised to  $C'_w (\neq C'_\infty)$  or a solute is supplied at a constant rate, and it is assumed that the heat transfer from the surface is proportional to the local surface temperature T'. As the plate is infinite in the x'-direction, all the physical variables are independent of x' and are functions of y' and t'only. Then under usual Boussinesq's approximation, after neglecting the inertia terms, viscous dissipation heat and Soret-Dufour effects, the flow can be shown to be governed by the following system of equations:

$$\frac{\partial u'}{\partial t'} = v \frac{\partial^2 u'}{\partial {y'}^2} + g\beta(T' - T'_{\infty}) + g\beta^*(C' - C'_{\infty})$$
(1)

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial {v'}^2} - \frac{\partial q_r}{\partial {v'}}$$
(2)

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial {v'}^2} \tag{3}$$

with the initial and boundary conditions

$$t' \leq 0: u' = 0, \ T' = T'_{\infty}, \ C' = C'_{\infty} \quad \text{for} \quad y' \geq 0,$$

$$t' > 0: \begin{cases} u' = U_0 f(t'), \ \frac{\partial T'}{\partial y'} = -\frac{h}{k}T', \\ C' = C'_w (or) \ \frac{\partial C'}{\partial y'} = -\frac{j''}{D} \end{cases} \text{ at } y' = 0, \end{cases}$$

$$(4)$$

$$u' \to 0, T' \to T'_{\infty}, C' \to C'_{\infty} \text{ as } y' \to \infty.$$

where u' is the fluid velocity in the x'-direction, T'and C' are the temperature and concentration of the fluid near the plate, v is the kinematic viscosity, gacceleration due to gravity,  $\beta$  is the volumetric coefficient of thermal expansion,  $\beta^*$  is the volumetric coefficient of concentration expansion,  $\rho$  is the density of the ambient fluid,  $C_p$  is the specific heat of the fluid at constant pressure, k is the thermal conductivity,  $q_r$  is the component of radiative flux, his the heat transfer coefficient, D is the mass diffusivity and j'' is the mass flux at the plate. The radiative heat flux term is simplified by making use of the Rosseland approximation<sup>24, 25</sup> as

$$q_r = -\frac{4\sigma}{3K_R} \frac{\partial T'^4}{\partial y'} \tag{5}$$

It should be noted that by using the Rosseland approximation we limit our analysis to optically thick fluids. If temperature differences within the flow are sufficiently small such that  $T'^4$  may be expressed as a linear function of the temperature, then the Taylor series for  $T'^4$  about  $T'_{\infty}$ , after neglecting higher order terms, is given by

$$T'^{4} \cong 4T'_{\infty}{}^{3}T' - 3T'_{\infty}{}^{4} \tag{6}$$

In view of Eqs. (5) and (6), Eq. (2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial {y'}^2} + \frac{16\sigma T_{\infty}^{\prime 3}}{3K_R} \frac{\partial^2 T'}{\partial {y'}^2}$$
(7)

#### **Uniformly Accelerated Plate**

For this case f(t') = t' and introducing the following non-dimensional quantities:

$$y = \frac{y'h}{k}, t = \frac{t'vh^2}{k^2}, u = \frac{u'vh^2}{U_0k^2Gr},$$

$$\theta = \frac{T'-T'_{\infty}}{T'_{\infty}}, \Pr = \frac{\mu C_p}{k}, Gr = \frac{g\beta T'_{\infty}}{U_0},$$

$$R = \frac{kK_R}{4\sigma T'_{\infty}}, C = \frac{C'-C'_{\infty}}{C'_w - C'_{\infty}} (UWC),$$

$$C = \frac{C'-C'_{\infty}}{(j''k/Dh)} (UMF), N = \frac{Gm}{Gr},$$

$$Gm = \frac{g\beta^*(C'_w - C'_{\infty})}{U_0} (UWC),$$

$$Gm = \frac{g\beta^*j''k}{U_0Dh} (UMF), Sc = \frac{v}{D}.$$
(8)

The corresponding dimensionless forms of governing equations are

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial v^2} + \theta + NC \tag{9}$$

$$A\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial v^2}$$
, where  $A = \frac{3R \operatorname{Pr}}{3R+4}$  (10)

$$Sc\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2}$$
(11)

The transformed initial and boundary conditions are

$$t \le 0: \ u = 0, \ \theta = 0, \ C = 0 \quad \text{for} \quad y \ge 0,$$
$$\left\{ \begin{aligned} u &= \frac{t}{Gr}, \ \frac{\partial \theta}{\partial y} = -(\theta + 1), \\ C &= 1 \left( \text{or} \right) \frac{\partial C}{\partial y} = -1 \\ u \to 0, \ \theta \to 0, \ C \to 0 \quad \text{as} \quad y \to \infty. \end{aligned} \right\}$$
(12)

where u,  $\theta$  and C are the dimensionless velocity, temperature and concentration, respectively, Pr is the Prandtl number, Gr is the thermal Grashof number, R is the radiation parameter, Sc is the Schmidt number, N is the buoyancy ratio parameter, and Gmis the mass Grashof number. The partial differential equations (9), (10) and (11) are solved subject to the initial and boundary conditions (12) by the Laplace transform technique. The solutions for the temperature and concentration fields are well known<sup>25</sup> and the solutions for the velocity field are given as follows:

Case 1:  $Pr \neq 1, Sc \neq 1$ 

$$u(y,t) = \frac{1}{Gr} f_{3}(y,t) + \frac{A}{(A-1)} \Big[ f_{1}(y\sqrt{A},t) - f_{1}(y,t) - f_{4}(y\sqrt{A},t) + f_{4}(y,t) \Big] + \frac{\sqrt{A}}{(A-1)} \Big[ f_{2}(y\sqrt{A},t) - f_{2}(y,t) \Big]$$
(13a)  
+  $\frac{1}{(A-1)} \Big[ f_{3}(y\sqrt{A},t) - f_{3}(y,t) \Big] + \frac{N}{(Sc-1)} \Big[ f_{3}(y,t) - f_{3}(y\sqrt{Sc},t) \Big]$ (UWC)

$$u(y,t) = \frac{1}{Gr} f_{3}(y,t) + \frac{A}{(A-1)} \Big[ f_{1}(y\sqrt{A},t) - f_{1}(y,t) - f_{4}(y\sqrt{A},t) + f_{4}(y,t) \Big] + \frac{\sqrt{A}}{(A-1)} \Big[ f_{2}(y\sqrt{A},t) - f_{2}(y,t) \Big]$$
(13b)  
$$+ \frac{1}{(A-1)} \Big[ f_{3}(y\sqrt{A},t) - f_{3}(y,t) \Big] + \frac{N}{\sqrt{Sc}(Sc-1)} \Big[ f_{5}(y,t) - f_{5}(y\sqrt{Sc},t) \Big] (UMF)$$

Case 2:  $Pr \neq 1, Sc = 1$ 

$$u(y,t) = \frac{1}{Gr} f_{3}(y,t) + \frac{A}{(A-1)} \Big[ f_{1}(y\sqrt{A},t) - f_{1}(y,t) - f_{4}(y\sqrt{A},t) + f_{4}(y,t) \Big] + \frac{\sqrt{A}}{(A-1)} \Big[ f_{2}(y\sqrt{A},t) - f_{2}(y,t) \Big] + \frac{1}{(A-1)} \Big[ f_{3}(y\sqrt{A},t) - f_{3}(y,t) \Big] + \frac{N y}{2} f_{2}(y,t) \text{ (UWC)}$$
(13c)

$$u(y,t) = \frac{1}{Gr} f_{3}(y,t) + \frac{A}{(A-1)} \Big[ f_{1}(y\sqrt{A},t) - f_{1}(y,t) - f_{4}(y\sqrt{A},t) + f_{4}(y,t) \Big] + \frac{\sqrt{A}}{(A-1)} \Big[ f_{2}(y\sqrt{A},t) - f_{2}(y,t) \Big] + \frac{1}{(A-1)} \Big[ f_{3}(y\sqrt{A},t) - f_{3}(y,t) \Big] + \frac{N}{2} f_{3}(y,t) \text{ (UMF)}$$

$$(13d)$$

where 
$$A = \frac{3R \operatorname{Pr}}{3R + 4}$$
,  
 $f_1(z,t) = \operatorname{erfc}\left(\frac{z}{2\sqrt{t}}\right)$ ,  
 $f_2(z,t) = 2\sqrt{t/\pi} \exp\left(-\frac{z^2}{4t}\right) - z \operatorname{erfc}\left(\frac{z}{2\sqrt{t}}\right)$ ,  
 $f_3(z,t) = \left(\frac{z^2}{2} + t\right) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}}\right)$ ,  
 $-z\sqrt{t/\pi} \exp\left(-\frac{z^2}{4t}\right)$ ,  
 $f_4(z,t) = \exp\left(\frac{t}{A} - \frac{z}{\sqrt{A}}\right) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}} - \sqrt{\frac{t}{A}}\right)$ ,  
 $f_5(z,t) = \frac{1}{3}(z^2 + 4t)\sqrt{t/\pi} \exp\left(-\frac{z^2}{4t}\right)$ ,  
 $-\left(\frac{z^3}{6} + zt\right) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}}\right)$ ,

z is a dummy variable and  $f_1, f_2, f_3, f_4, f_5$  are dummy functions.

From the velocity field, it is interesting to study the effects of the system parameters on the skin friction. It is given by

$$\tau = \frac{\tau' h}{\rho U_0 k} = -\frac{\partial u}{\partial y} \bigg|_{y=0}$$
(14)

From Eqs. (13) and (14), we have

$$\tau = \frac{2\sqrt{t}}{Gr\sqrt{\pi}} + \frac{\sqrt{A}}{\sqrt{A}+1} \left[ 1 + 2\sqrt{\frac{t}{A\pi}} - \exp\left(\frac{t}{A}\right) \left(1 + \exp\left(\frac{t}{A}\right) \left(1 + \exp\left(\frac{t}{A}\right)\right) \right]$$
(15a)
$$-\frac{2N\sqrt{t}}{\sqrt{\pi}(\sqrt{Sc}+1)} (UWC)$$

$$\tau = \frac{2\sqrt{t}}{Gr\sqrt{\pi}} + \frac{\sqrt{A}}{\sqrt{A}+1} \left[ 1 + 2\sqrt{\frac{t}{A\pi}} - \exp\left(\frac{t}{A}\right) \left( 1 + \exp\left(\frac{t}{A}\right) \right) \right]$$
(15b)  
$$- \frac{Nt}{\sqrt{Sc}(\sqrt{Sc}+1)} (UMF)$$

## **Exponentially Accelerated Plate**

For this case,  $f(t') = \exp(a't')$ , where a' is the exponentially accelerating parameter and the velocity field in the dimensionless form is given by

Case 1:  $Pr \neq 1, Sc \neq 1$ 

$$\begin{split} u(y,t) &= \frac{\exp(at)}{Gr} f_6(y,t) \\ &+ \frac{A}{(A-1)} \Big[ f_1(y\sqrt{A},t) - f_1(y,t) \\ &- f_4(y\sqrt{A},t) + f_4(y,t) \Big] \\ &+ \frac{\sqrt{A}}{(A-1)} \Big[ f_2(y\sqrt{A},t) - f_2(y,t) \Big] \\ &+ \frac{1}{(A-1)} \Big[ f_3(y\sqrt{A},t) - f_3(y,t) \Big] \\ &+ \frac{N}{(\mathrm{Sc}-1)} \Big[ f_3(y,t) - f_3(y\sqrt{\mathrm{Sc}},t) \Big] (\mathrm{UWC}) \end{split}$$

where

$$f_6(z,t) = \frac{1}{2} \left[ \exp\left(-z\sqrt{a}\right) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}} - \sqrt{at}\right) + \exp\left(z\sqrt{a}\right) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}} + \sqrt{at}\right) \right]$$

is a dummy function.

$$u(y,t) = \frac{\exp(at)}{Gr} f_{6}(y,t) + \frac{A}{(A-1)} \Big[ f_{1}(y\sqrt{A},t) - f_{1}(y,t) - f_{4}(y\sqrt{A},t) + f_{4}(y,t) \Big] + \frac{\sqrt{A}}{(A-1)} \Big[ f_{2}(y\sqrt{A},t) - f_{2}(y,t) \Big]$$
(16b)  
$$+ \frac{1}{(A-1)} \Big[ f_{3}(y\sqrt{A},t) - f_{3}(y,t) \Big] + \frac{N}{\sqrt{\operatorname{Sc}(\operatorname{Sc}-1)}} \Big[ f_{5}(y,t) - f_{5}(y\sqrt{\operatorname{Sc}},t) \Big] (\operatorname{UMF})$$

Case 2:  $Pr \neq 1, Sc = 1$ 

$$u(y,t) = \frac{\exp(at)}{Gr} f_{6}(y,t) + \frac{A}{(A-1)} \Big[ f_{1}(y\sqrt{A},t) - f_{1}(y,t) - f_{4}(y\sqrt{A},t) + f_{4}(y,t) \Big] + \frac{\sqrt{A}}{(A-1)} \Big[ f_{2}(y\sqrt{A},t) - f_{2}(y,t) \Big]$$
(16c)  
$$+ \frac{1}{(A-1)} \Big[ f_{3}(y\sqrt{A},t) - f_{3}(y,t) \Big] + \frac{N y}{2} f_{2}(y,t) (UWC) u(y,t) = \frac{\exp(at)}{Gr} f_{6}(y,t) + \frac{A}{(A-1)} \Big[ f_{1}(y\sqrt{A},t) - f_{1}(y,t) - f_{4}(y\sqrt{A},t) + f_{4}(y,t) \Big] + \frac{\sqrt{A}}{(A-1)} \Big[ f_{2}(y\sqrt{A},t) - f_{2}(y,t) \Big]$$
(16d)  
$$+ \frac{1}{(A-1)} \Big[ f_{3}(y\sqrt{A},t) - f_{3}(y,t) \Big] + \frac{N y}{2} f_{3}(y,t) (UMF)$$

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The non-dimensional quantities used in Eqs. (16) are defined as follows:

$$y = \frac{y'U_{0}}{v}, t = \frac{t'U_{0}^{2}}{v}, u = \frac{u'}{U_{0}Gr},$$

$$\theta = \frac{T' - T'_{\infty}}{T'_{\infty}}, \Pr = \frac{\mu C_{p}}{k}, Gr = \frac{vg\beta T'_{\infty}}{U_{0}^{3}},$$

$$a = \frac{a'v}{U_{0}^{2}}, R = \frac{k K_{R}}{4\sigma T'_{\infty}^{3}}, C = \frac{C' - C'_{\infty}}{C'_{w} - C'_{\infty}} (UWC),$$

$$C = \frac{C' - C'_{\infty}}{(j''v / U_{0}D)} (UMF), N = \frac{Gm}{Gr},$$

$$Gm = \frac{vg\beta^{*}(C'_{w} - C'_{\infty})}{U_{0}^{3}} (UWC),$$

$$Gm = \frac{v^{2}g\beta^{*}j''}{DU_{0}^{4}} (UMF), Sc = \frac{v}{D}.$$
(17)

where *a* is the dimensionless exponentially accelerating parameter. The skin-friction at an exponentially accelerated vertical plate is given by

$$\tau = \frac{\tau'}{\rho U_0^2 G r} = -\frac{\partial u}{\partial y}\Big|_{y=0}$$
(18)

From Eqs. (16) and (18), it can be shown that

$$\tau = \frac{1}{Gr\sqrt{\pi t}} + \frac{\sqrt{a} \exp(at)}{Gr} \operatorname{erf}\left(\sqrt{at}\right) + \frac{\sqrt{A}}{\sqrt{A}+1} \left[1 + 2\sqrt{\frac{t}{A\pi}}\right]$$
(19a)  
$$- \exp\left(\frac{t}{A}\right) \left(1 + \operatorname{erf}\left(\sqrt{\frac{t}{A}}\right)\right) - \frac{2N\sqrt{t}}{\sqrt{\pi}(\sqrt{Sc}+1)} (UWC)$$
(19a)  
$$\tau = \frac{1}{Gr\sqrt{\pi t}} + \frac{\sqrt{a} \exp(at)}{Gr} \operatorname{erf}\left(\sqrt{at}\right) + \frac{\sqrt{A}}{\sqrt{A}+1} \left[1 + 2\sqrt{\frac{t}{A\pi}}\right] (19b) - \exp\left(\frac{t}{A}\right) \left(1 + \operatorname{erf}\left(\sqrt{\frac{t}{A}}\right)\right) \right]$$
(19b)  
$$- \frac{Nt}{\sqrt{Sc}(\sqrt{Sc}+1)} (UMF)$$

#### **RESULTS AND DISCUSSION**

The representative values for the velocity and skinfriction are presented to illustrate the effects of N, Rand a when t = 0.2, Pr = 0.71 (air), Gr = 1 and Sc = 0.6 (water vapor). The velocity profiles are shown in Figure 1 for a uniformly accelerated plate at various values of the buoyancy ration parameter N for both the cases of UWC and UMF. It is seen from this figure that an increase in N leads to a rise in the fluid velocity due to increasing mass buoyancy force in both cases.



**FIGURE 1.** Velocity Profiles at Various *N* (Uniformly Accelerated Plate)



**FIGURE 2.** Velocity Profiles at Various *R* (Uniformly Accelerated Plate)

Figure 2 shows the velocity profiles at various R values for both UWC and UMF cases for a uniformly accelerated plate. It is observed that an increase in the radiation parameter value leads to a decrease in fluid velocity. The reason is that an increase in the radiation parameter means an increase in the radiation

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absorption coefficient. It is clear from Figures 1 and 2 that the fluid velocity in the case of UWC is greater than the fluid velocity in the case of UMF at an early time t = 0.2.

Figure 3 shows the velocity variation with buoyancy ratio parameter (N) for an exponentially accelerated plate for both UWC and UMF cases. It can be seen that the velocity increases with increasing values of N. Figure 4 demonstrates the velocity variation at various values of radiation parameter (R) for an exponentially accelerated plate for both UWC and UMF cases. It is observed that the fluid velocity decreases with increasing value of R. The reason is same as mentioned before in the case of uniformly accelerated plate.



**FIGURE 3.** Velocity Profiles at Various *N* (Exponentially Accelerated Plate)



**FIGURE 4.** Velocity Profiles at Various *R* (Exponentially Accelerated Plate)

The effect of exponentially accelerating parameter (a) on the fluid velocity is demonstrated in Figure 5 for both UWC and UMF cases. This figure indicates

that the fluid velocity increases with increasing *a*. From Figures 3 to 5 it is clear that the velocity is slightly greater in the case of UWC than that of UMF at an early time t = 0.2. But this difference is not significant when compared with the case of uniformly accelerated plate.



**FIGURE 5.** Velocity Profiles at Various *a* (Exponentially Accelerated Plate)

The computed values of skin-friction  $(\tau)$  at various values of t, R, N and a are displayed in Table I for both uniformly accelerated plate and exponentially accelerated plate in both UWC and UMF cases. It can be seen that the skin-friction decreases with increasing time for all the cases. The value of the skin-friction become negative after some time, this indicates that reverse type of flow occurs near the accelerating plate as time progresses. The skin-friction increases with increasing R whereas it decreases with increasing Nfor all the cases. The skin-friction at an exponentially accelerated plate increases with increasing a at an early time t = 0.2. It is interesting to note that the skin-friction is greater at an exponentially accelerated plate than that of uniformly accelerated plate for both UWC and UMF boundary conditions at an early time.

#### **CONCLUSIONS**

An exact analysis of the unsteady free convection flow near an accelerated infinite vertical plate with Newtonian heating has been carried out for the uniform wall concentration (UWC) and uniform mass flux (UMF) boundary conditions in the presence of thermal radiation. The governing dimensionless partial differential equations have been solved analytically without any restrictions using the Laplace transform technique. It is observed that the fluid velocity increases with increasing buoyancy ratio parameter and it decreases with increasing radiation parameter in both UWC and UMF cases. The fluid velocity is greater in the case of UWC than the case of UMF for both uniformly accelerated plate and exponentially accelerated plate at an early time. But this difference is not significant in the case of exponentially accelerated plate as compared to that of uniformly accelerated plate. The skin-friction is greater at an exponentially accelerated plate than that of uniformly accelerated plate at an early time.

<b>TABLE 1.</b> Skin-friction	$(\tau)$	) Variation w	hen Pr =	0.71.	Gr = 1	l and Sc =	0.6.
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t	R	N	a	Uniformly Ac	celerated Plate	Exponentially A	Exponentially Accelerated Plate			
-				UWC	UMF	UWC	UMF			
0.1	10	0.2	0.5	0.217533	0.243198	1.829313	1.854978			
0.2	10	0.2	0.5	0.213236	0.241009	1.240003	1.267775			
0.4	10	0.2	0.5	0.016368	0.038599	0.603216	0.625446			
0.6	10	0.2	0.5	-0.405802	-0.394594	-0.015601	-0.004393			
0.8	10	0.2	0.5	-1.093548	-1.096201	-0.808597	-0.811250			
0.2	1	0.2	0.5	-0.070882	-0.043109	0.955884	0.983657			
0.2	5	0.2	0.5	0.184660	0.212433	1.211426	1.239199			
0.2	$\infty$	0.2	0.5	0.241244	0.269017	1.268010	1.295783			
0.2	10	0	0.5	0.270108	0.270108	1.296875	1.296875			
0.2	10	0.6	0.5	0.099492	0.182810	1.126258	1.209576			
0.2	10	1	0.5	-0.014253	0.124611	1.012514	1.151378			
0.2	10	0.2	0	-	-	0.970176	0.997949			
0.2	10	0.2	1.0	-	-	1.547790	1.575563			
0.2	10	0.2	1.5	-	-	1.898337	1.926110			

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