

# **A Comprehensive Study on the Current Pressure Drop Calculation in Multiphase Vertical Wells; Current Trends and Future Prospective**

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## **ABSTRACT**

A reliable estimation of the pressure drop in well tubing is essential for the solution of a number of important production engineering and reservoir analysis problems. Many empirical correlation and mechanistic models have been proposed to estimate the pressure drop in vertical wells that produce a mixture of oil, water and gas. Although many correlations and models are available to calculate the pressure loss, these models developed based on certain assumption and for particular range of data where it may not be applicable to be used in different sets of data. This paper presents an investigation on the predictive performance evaluation for the reliable methods used to calculate the pressure drop in multiphase vertical wells taking into consideration the dimensions of each model. Most correlations and models created to calculate pressure drop were developed based on accurately and reliably measured flow parameters. However, it can only work best on the proposed data range. Statistical error analysis and graphical error analysis are used to analyze the variation between predicted values and actual ones. Hence, it showed most reliable methods that can perform well in different well conditions. Based on the analysis of this study, the artificial neural networks models had showed better prediction accuracy and minimum number of variables even if other data beyond the range of data is used.

## **INTRODUCTION**

Multiphase flow in pipes is the process of simultaneous flow of two phases or more. In oil or gas production wells the multiphase flow usually consist of oil, gas and water. The estimation of the pressure drop in vertical wells is quite important for cost effective design of well completions, production optimization and surface facilities. However, due to the

complexity of multiphase flow several approaches have been used to understand and analysis the multiphase flow.

Oil & Gas industry is needed to have a general method for forecasting and evaluating the multiphase flow in vertical pipes (Poettmann, & Carpenter, 1952). Multiphase flow correlations are used to determine the pressure drop in the pipes. Although, many correlation and models have been proposed to calculate pressure drop in vertical well, yet it's still arguing about the effectiveness of these proposed models.

Numerous correlations and equations have been proposed for multiphase flow in vertical, inclined and horizontal wells in the literature. Early methods treated the multiphase flow problem as the flow of a homogeneous mixture of liquid and gas. This approach completely disregarded the well-known observation that the gas phase, due to its lower density, overtakes the liquid phase resulting in "slippage" between the phases. Slippage increases the flowing density of the mixture as compared to the homogeneous flow of the two phases at equal velocities. Because of the poor physical model adopted, calculation accuracy was low for those early correlations. Another reason behind that is the complexity in multiphase flow in the vertical pipes. Where water and oil may have nearly equal velocity, gas have much greater one. As a results, the difference in the velocity will definitely affect the pressure drop.

Many methods have been proposed to estimate the pressure drop in vertical wells that produce a mixture of oil and gas. The study conducted by Pucknell et al. (1993) concludes that none of the traditional multiphase flow correlations works well across the full range of conditions encountered in oil and gas fields. Besides, most of the vertical pressure drop calculation models were developed for average oilfield fluids and this is why special conditions such as; emulsions, non-Newtonian flow behavior, excessive scale or wax

deposition on the tubing wall, etc. can pose severe problems. Accordingly, predictions in such cases could be doubtful. (Takacs, 2001)

The early approaches used the empirical correlation methods such as Hagedron & Brown (1965) Duns & Ros, (1963), and Orkiszewski (1967). Then the trend shift into mechanistic modelling methods such as Ansari (1994) and Aziz et al (1972) and lately the researchers have introduced the use of artificial intelligence into the oil and gas industry by using artificial neural networks such as Ayoub (2004) and Mohammadpoor (2010) and many others.

The main purpose of this study is to evaluate and assess the current empirical correlations, mechanistic model and artificial neural networks for pressure drop estimation in multiphase flow in vertical wells by comparing the most common methods in this area. The parameters affecting the pressure drop are very important for the pressure calculation. Therefore, it will also be taken into account in the evaluation.

## **EMPIRICAL CORRELATIONS**

The empirical correlation was created by using mathematical equations based on experimental data. Most of the early pressure drop calculation was based on this correlations because of its direct applicability and fair accuracy to the data range used in the model generation. In this study, the empirical correlations for pressure drop estimation in multiphase flow in vertical wells are reviewed and evaluated with consideration of its required dimensions, performance, limitation and range of applicability.

**Duns & Ros Correlation (1963):** This empirical correlation is resulted from laboratory experiments with some modification and adjustments in the correlation by using

actual field data. Duns & Ros correlation is in terms of a dimensionless gas velocity number, diameter number, liquid velocity number and a dimensionless mathematical expression. The acceleration gradient is neglected in the methods. Although this method is developed to calculate the pressure drop with dry oil/gas mixtures, it can also be used with wet oil/gas mixtures in some cases.

**Hagedron & Brown Correlation (1965):** Hagedron & Brown correlation is one of the most common correlations used in the industry. Hagedron & Brown correlation developed using an experimental study of pressure gradients occurring during continuous two-phase flow in small diameter vertical conduits, a 1500 ft. vertical wellbore and considering 5 different fluids types in the experiment which is water and four types of oil. This correlation involves only dimensionless groups of variables and it can be applied over a much wider range of conditions compared to other correlations.

**Orkiszewski Correlation (1967):** This correlation had developed an equation for two-phase pressure drops in flowing and gas-lift production wells over a wide range of well conditions with range of precision about 10%. The method is an extension of the work done by Griffith and Wallis (1961). The correlation is valid for several flow regimes such as; bubble flow, slug flow, transition flow and annular-mist flow. Orkiszewski proved his assumptions by comparing the measured pressure drop results of 184 wells to the calculated ones. The parameters considered in his equation for the pressure drop are the effect by the energy lost by friction, the change in potential energy and the change in kinetic energy. The results obtained by these methods are still applicable for wide range of well conditions (e.g. heavy oil). But, there are some well conditions that have not been evaluated (e.g., flow in the casing annulus and in the mist flow regime).

**Beggs & Brill Correlation (1973):** The Beggs and Brill method was developed to predict the pressure drop for horizontal, inclined and vertical flow. It also takes into account the several flow regimes in the multiphase flow. Therefore, Beggs & Brill (1973) correlation is the most widely used and reliable one by the industry. In their experiment, they used 90 ft. long acrylic pipes data. Fluids used were air and water and 584 tests were conducted. Gas rate, liquid rate and average system pressure was varied. Pipes of 1 and 1.5 inch diameter were used. The parameters used are gas flow rate, Liquid flow rate, pipe diameter, inclination angle, liquid holdup, pressure gradient and horizontal flow regime. This correlation has been developed so it can be used to predict the liquid holdup and pressure drop.

**Gray Correlation (1978):** The Gray correlation was developed by H.E. Gray, specifically for wet gas wells. Although this correlation was developed for wet gas vertical flow, but it can also be used in multiphase vertical and inclined flow. In his correlation flow is treated as single phase, and dropped out water or condensate is assumed to adhere to the pipe wall. The parameters considered in this method are the phase velocity, tube size gas condensate ratio and water ratio. The pressure difference due to friction is calculated using the Fanning friction pressure loss equation.

**Mukherjee & Brill Correlation (1985):** Mukherjee & Brill Proposed a correlation for Pressure loss, Holdup and flow map. Their correlation was developed following a study of pressure drop behaviour in two-phase inclined flow. However, it can also be applied to vertical flow. Prior knowledge of the liquid holdup is needed to compute the pressure drop using Mukherjee & Brill (1985) correlation. The results obtained from their experiments were verified with Prudhoe Bay and North Sea data.

## MECHANISTIC MODELS

Mechanistic models or known also as semi-empirical correlations deal with the physical phenomena of the multiphase flow. These kinds of models are developed by using mathematical modelling approach. A fundamental hypothesis in this type of models is the existence of various flow configurations or flow patterns, including stratified flow, slug flow, annular flow, bubble flow, churn flow and dispersed bubble flow. The first objective of this approach is thus, to predict the existing flow pattern for a given system. Although most of the current presented mechanistic models have been developed under certain conditions which limit their ability to be used in different range of data, these models are expected to be more reliable and general because they incorporate the mechanisms and the flow important parameters (Gomez et al. 2000).

**Aziz et al. Model (1972):** Aziz, Govier and Fogarasi (1972) have proposed a simple mechanistically based scheme for pressure drop calculation in wells producing oil and gas. The scheme was based on the identification of the flow pattern map. The mechanical energy equation was presented in the relationship between the pressure gradient, the flow rate, the fluid properties and the geometry of the flow duct. While the model proposed new equations for bubble and slug flow patterns, it recommended the old Dun & Ros equations for annular mist pattern. The new prediction method incorporates an empirical estimation of the distribution of the liquid phase between that flowing as a film on the wall and that entrained in the gas core. It employs separate momentum equations for the gas-liquid mixture in the core and for the total contents of the pipe. The model has presented 44 value of predicted pressure drop with absolute error almost equal to that for Orkiszewski correlation. However,

the uncertainties and lack of some field data made it difficult to develop a fully mechanistically, reliable based computation method.

**Ansari et al. Model (1994):** This mechanistic model is developed for upward two-phase flow in wellbores. This model was developed as part of the Tulsa University Fluid Flow Projects (TUFFP) research program. The model predicted the existence of four flow patterns which are; bubble flow, slug flow, churn flow and annular flow. The model was evaluated by using the TUFFP well databank that is composed of 1775 well cases, with 371 of them from Prudhoe Bay data. Ansari et al (1994) claimed that the overall performance of the comprehensive model is superior to all other methods considered with an exception of Hagedorn & Brown empirical correlation due to extensive data used in its development and modifications made to the correlation.

## **ARTIFICIAL NEURAL NETWORKS**

An artificial neural networks model is a structure (network) composed of a number of interconnected units (artificial neurons). Each unit has an input/output (I/O) characteristic and implements a local computation or function (Jahanandish & Jalalifar, 2011). It has been only a few years since neural networks first gained popularity. In the past two to three years banks, credit card companies, manufacturing companies, high tech companies and many more institutions have adopted neural nets to help in their day-to-day operation. Most researchers believe that artificial neural networks may be able to produce what rule based artificial intelligence (expert systems) have promised for so long but failed to deliver.

The use of Artificial Neural Networks (ANNs) in petroleum industry can be tracked back several years ago. The literature has many industry problems solved by several authors using ANNs models. ANNs have been used in several area of oil and gas industry such as; permeability prediction, well testing, enhanced oil recovery, PVT properties prediction,



improvement of gas well production, prediction and optimization of well performance and integrated reservoir characterization and portfolio management. (Ayoub, 2004).

Experience showed that empirical correlations and mechanistic models failed to provide a satisfactory and reliable tool for estimating pressure drop in multiphase flowing wells. Large errors are usually associated with these models and correlations (Takacs, 2001). Artificial neural networks gained wide popularity in solving difficult and complex problems, especially in petroleum engineering (Mohaghegh and Ameri, 1995).

**Ayoub Model (2007):** Ayoub presented an Artificial Neural Networks (ANNs) model for prediction bottom-hole flowing pressure and consequently the pressure drop in vertical multiphase flow. The model was developed and tested using field data covering a wide range of variables. A total of 206 field data sets collected from Middle East fields; were used to develop the ANN model. These data sets were divided into training, cross validation and testing sets in the ratio of 3:1:1. The testing subset of data, which was not seen by the ANN model during the training phase, was used to test the prediction accuracy of the model. Trend analysis of the model showed that the model correctly predicted the expected effects of the independent variables on bottomhole flowing pressure. This indicated that the model simulates the actual physical process. Although, the results showed that his model significantly outperformed all existing methods and provided predictions with higher accuracy. The author claimed that his model can be used only within the range of used data. Consequently, caution should be taken beyond the range of used input variables. Ayoub (2004) model demonstrates the power of artificial neural networks model in solving complicated engineering problems.

## SOURCE OF DATA

A total of 260 data sets were collected from different Middle East fields and believed to be quite reliable. The data used for comparing the different pressure predicting methods covers an oil rate from 45 to 19618 BPD, water cut up to 91.8%, oil gravity from 13.60 to 37.00 API and wellhead pressure from 5 to 640 psia. All data sets which consistently resulted in poor predictions by all correlations and mechanistic models were considered to be invalid; hence, it has been removed. Table (1) shows the statistical analysis of the used data.

**Table 1: Statistical Analysis of the Used Data**

<b>Flow Parameter</b>	<b>Min</b>	<b>Max</b>	<b>Average</b>	<b>STD</b>
<b>Bottomhole Pressure, (psi)</b>	1019.79	3105.00	2205.30	486.66
<b>Oil Rate, (bbl/D)</b>	45.20	19618.00	4552.16	4598.26
<b>Water Rate, (bbl/D)</b>	0.00	7900.00	1784.68	2368.18
<b>Gas Rate, (Mscf/D)</b>	0.00	12495.00	2203.67	2755.98
<b>Depth, (ft)</b>	2726.38	8070.87	5810.17	1049.09
<b>Tubing Diameter, (in)</b>	2.00	4.00	3.75	0.33
<b>Surface Temperature, (degreeF)</b>	70.00	160.00	113.55	27.44
<b>Wellhead Pressure, (psi)</b>	5.00	640.00	240.81	148.88
<b>Oil Gravity, (API)</b>	13.60	37.00	30.77	5.85

## Selected Methods for Evaluation

Based on the extensive literature study carried out by the authors, six empirical correlations, from which two mechanistic models and one artificial neural networks model were selected for evaluation. These are:

- Ansari et al Model,

- Ayoub Model,
- Aziz et al Model,
- Beggs & Brill Correlation,
- Duns & Ros Correlation,
- Gray Correlation,
- Hagedron & Brown Correlation,
- Mukherjee & Brill Correlation,
- And Orkiszewski Correlation.

## **EVALUATION PROCESS**

The common obstacle for using a pressure drop method whether it's an empirical correlation, a mechanistic model or an artificial neural network model that most of these models are applicable for specific range of data and conditions in order to predict the pressured drop accurately. However, in some cases, it can work well also in some actual filed data with acceptable perdition error.

To analyse and compare the effectiveness of each correlation or model, the values of both measured and predicted pressure drop are recorded. All the selected correlations and models are evaluated using actual filed data where the predicted pressure drop is compared to the measured one. The analysis is conducted via statistical and graphical error analysis.

Statistical Error Analysis has been used to check the accuracy of the model. The statistical parameters used in this paper are average absolute percent relative error, average percent relative error, maximum absolute percentage error, minimum absolute percentage error, root mean square error, coefficient of determination and the standard deviation of error. Equations for those parameters are given in the appendix. To confirm the obtained results only one graphical error analysis used which is cross plot. Cross plots were used to compare

the performance of all the selected models. A 45° straight line between the calculated pressure drop values versus measured pressure drop values is plotted which represents a perfect correlation line. When the values go closer to the line, it indicates better agreement between the measured and the estimated values.

## RESULTS

The evaluation for each methods will be based on the bottomhole pressure prediction and then and the estimated pressure drop. Table (2) shows the average absolute percentage relative error, average percentage relative error, maximum absolute relative error, minimum absolute relative error, root mean square error, coefficient of determination and the Standard deviation of all the selected methods. Aziz et al model has achieved the highest average absolute percentage relative error with 12.1% while Ayoub model achieved the lowest value with 4.8 %. Figure (1) shows the absolute and relative error of each empirical correlation, mechanistic model and artificial neural networks.

Figure (2) through Figure (4) indicate the performance of all investigated models. Aziz et al mechanistic model achieved the worst average absolute relative error (AAPE), root mean square error (RMSE) and coefficient of determination ( $R^2$ ) among all investigated models while Ayoub model achieves the best results.

A close result can be extracted when root mean square error (RMSE) of each model has been plotted against the standard deviation (STD) of errors, as presented in Figure (5). The best model will be located at the lower left corner, which is indicated by the intersection of both lower values of RMSE and STD. As in this case, Ayoub model performed the best, followed by Beggs & Brill correlation, Dun & Ros correlation and Ansari et al. model.

**Table 2: Statistical Analysis Results of the Selected Methods**

<b>Method</b>	<b>AAPE</b>	<b>APE</b>	<b>MaxAPE</b>	<b>MinAPE</b>	<b>RMSE</b>	<b>R<sup>2</sup></b>	<b>STD</b>
<b>Aziz et al</b>	12.0968	5.8956	46.6863	0.1688	15.8240	0.5158	14.6847
<b>Hagedron &amp; Brown</b>	11.9864	10.3126	31.3833	0.2806	13.7535	0.8065	9.0999
<b>Gray</b>	11.8941	9.9174	50.6174	0.4964	14.3411	0.7875	10.3591
<b>Orkiszewski</b>	11.0000	8.7381	26.7816	0.0611	13.0893	0.7692	9.7455
<b>Mukherjee &amp; Brill</b>	9.1695	4.0663	39.3635	0.0004	11.4425	0.7981	10.6956
<b>Ansari et al</b>	7.6344	4.9474	24.2722	0.0475	9.5011	0.8442	8.1114
<b>Duns &amp; Ros</b>	7.5593	3.5755	30.0916	0.0851	9.3525	0.8537	8.6421
<b>Beggs &amp; Brill</b>	6.4278	2.1967	24.9539	0.0851	8.2240	0.8667	7.9252
<b>Ayoub</b>	4.8010	1.3000	20.1594	0.0150	6.6274	0.9095	6.4987

Also, average absolute relative error (AAPE) of each model has been plotted against the confident of determination ( $R^2$ ), as presented in Figure (6). However, this time the best model will be located at the upper left corner, which is indicated by the intersection of both low AAPE value with High  $R^2$ . As in this case, Ayoub model performed the best, followed by Beggs & Brill correlation, Dun & Ros correlation and Ansari et al. model.

Figure (7) through Figure (15) present cross-plots of estimated pressure drop versus measured pressure drop for each of the investigated methods. These figures show the difficulty and instability of estimating the pressure drop. It has been noticed that most of investigated methods, Duns & Ros (1963), Hagedron & Brown (1965), Orkiszewski (1967), Beggs & Brill (1973), Gray (1978), Mukherjee & Brill (1985) and Aziz et al. (1972) tend to underestimate the pressure drop value. On the other hand, Ansari et al. (1994) and Ayoub (2004) have showed a quite good correlation around the actual values.

## **CONCLUSIONS**

The main aim of this study is to compare the current available methods of predicting pressure drop in multiphase vertical wells, their accuracy, performance and applicability. However, there is no rule of thumb when it comes to choosing the best general method that can satisfy all conditions. Therefore, based on the previous results. Authors have reached the following conclusions:

- For very accurate prediction, the correlation that gives better result will vary depending on the data investigated and the physical situation.
- For easy and fast prediction, the artificial neural networks model has shown superiority to the empirical correlations and mechanistic models.
- Ayoub model and Beggs & Brill correlations showed good estimation while Aziz et al. model showed severe underestimation of the pressure drop.

## **RECOMMENDATIONS**

Based on the above conclusions, the authors would like to suggest the following recommendation:

- Ayoub (2004) artificial neural networks model is highly recommended for predicting pressure drop in vertical well in multiphase flow especially for the data range recommended by the model.
- Beggs & Brill Correlation (1973) is a good correlation and also recommended to be used when Ayoub (2004) model is not available.
- More improvements and developments in the artificial neural networks

models for predicting pressure drop in the multiphase vertical flowing wells will definitely lead to better and accurate prediction in the future. Hence, all focuses and researches are highly recommended to go through that direction.

Not to be forgotten, there are still many empirical correlations, mechanistic models and artificial neural networks in the literature which have not been evaluated in this study and may have more or less accurate results when predicting pressure drop in vertical wells. However, the methods were selected based on the authors' perspective. Therefore, all the conclusions and recommendations were based on the selected methods.

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## **REFERENCES**

- Abdul-Majeed, G. H., & Al-Mashat, A. M. (2000). A mechanistic model for vertical and inclined two phase slug flow. *Journal of Petroleum Science and Engineering*, 59-67.
- Al-Attar, H. H., Mohamed, M. Y., & Amin, M. E. (2012). A Modified Version of the Aziz et al. Multiphase Flow correlation Improves Pressure drop Calculation in High-Rate Oil wells. *EAGE annual conference and exhibition, 4-7 June 2012*. Copenhagen, Denmark.
- Ansari, A. M., Sylvester, N. D., Sarica, C., Shoham, O., & Brill, J. P. (1994). A Comprehensive Mechanistic Model for Upward Two-Phase Flow in Wellbore. *SPE Production and Facilities*, May 1994, 143-152.

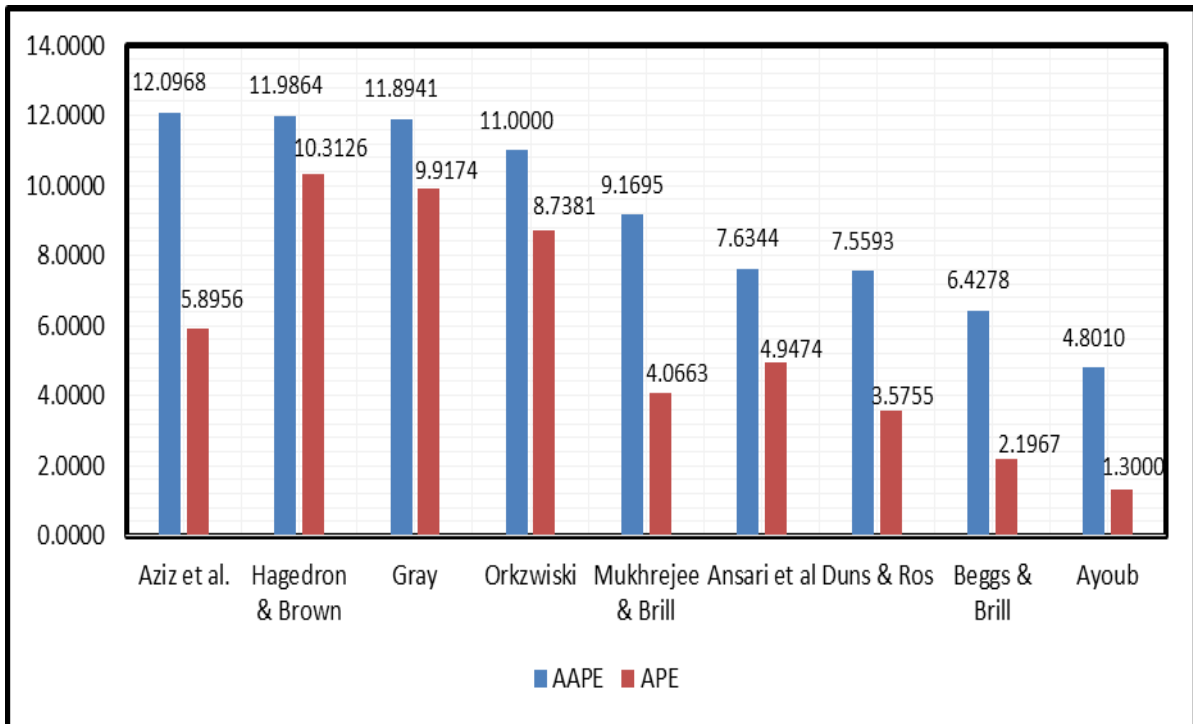
- Ayoub , M. A. (2011). *Development and Testing of Universal Pressure Drop Model in Pipelines Using Abductive and Artificial Neural Networks*. Bandar Seri Iskander, Perak: Phd Thesis, Universiti Teknologi Petronas.
- Ayoub, M. A. (March 2004). *Development and Testing of an Artificial Neural Network Model for Predicting Bottomhole Pressure in Vertical Multiphase Flow*. Dahrhan, Saudi Arabia: M S Thesis, King Fahd University of Petroleum and Minerals.
- Aziz, K., Govier, G. W., & Fogarasi, M. (1972). Pressure Drop in Wells producing Oil and Gas. *The Journal of Canadian Petroleum*, 38-48.
- Beggs, H. D., & Brill, J. P. (May 1973). A Study in Two-Phase Flow in Inclined Pipes. *Journal of Petroleum Technology*, 607-17., AIME 255.
- Chierici, G. L., Ciucci, G. M., & Sclocchi, G. (1974). Two-Phase Vertical Flow in Oil Wells- Prediction of Pressure drop. *Journal of Petroleum Technology*, August 1974, 927-938.
- Dun, R., & Ros, N. C. (1963). VERTICAL FLOW OF GAS AND LIQUID MIXTURES IN WELLS. *Proc., Sixth World Pet. Con- gress, Frankfort (J nne 19-26, 1963) Section II, Paper 22-PD6.*, (pp. 451-465).
- Gomez, L. E., Shoham, O., Schmidt, Z., Chokshi, R. N., & Northug, T. (2000). Unified Mechanistic Model Foe Steady State Two-Phase Flow: Horizontal to Vertical Upward Flow. *SPE Journal*, Vol. 5, No. 3, September , 393-350.
- Gould, T. L., Tek, M. R., & Katz, D. L. (1974). Two-Phase Flow Through Vertical, Inclined or Curved Pipe. *Journal of Petroleum Technology*, 915-926.
- Govier, G., & Aziz, k. (1972). *The Flow of Complex Mixtures in Pipes*. Van Nostrand-Reinhold, New York.



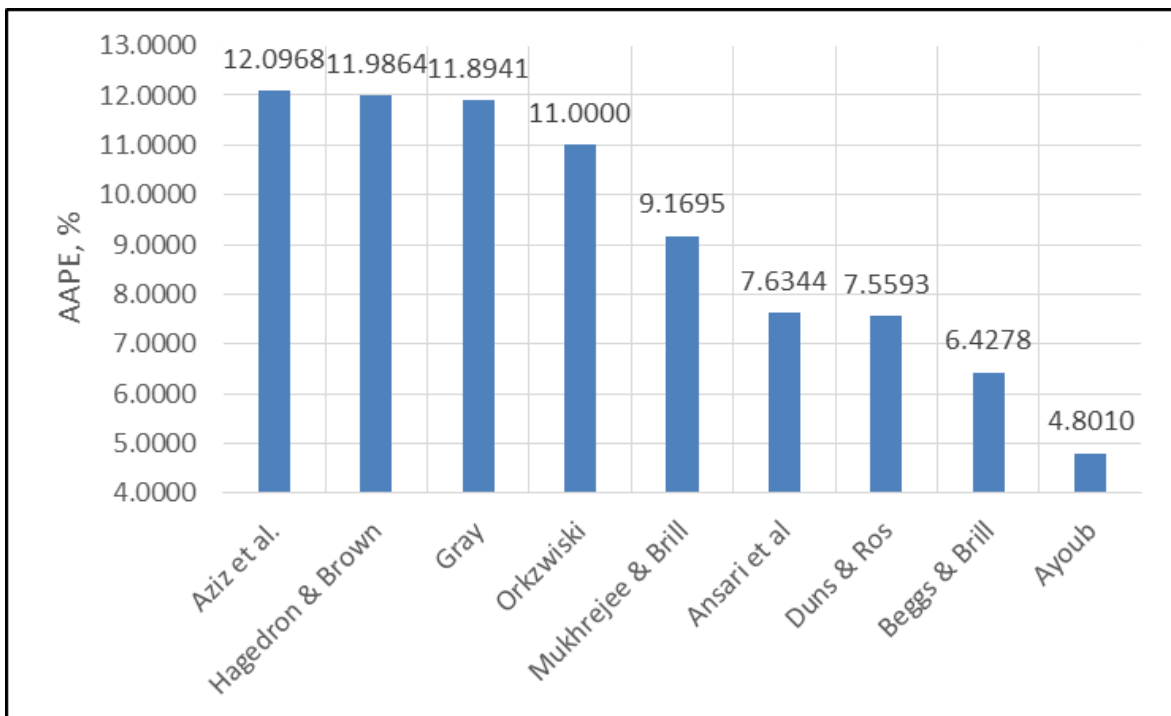
- Gregory, G. A. (1974). Application of Factorial Design Analysis to Producing Well Pressure-Drop Modelling. *The Journal of Canadian Petroelum Technology*, 21-27.
- Griffith, P., & Wallis, G. B. (1961). Two-Phase Slug Flow. *Journal of heat Transefer, A.S.M.E transaction (AUG 1961) Vol.83*, 307-320.
- Griffith, P., Lau, C. W., Hon, P. C., & Pearson, J. F. (1975). Two Phase Pressure Drop in Inclined and Vertical Wells.
- Hagedron, A., & Brown, K. (1965). Experimental study of pressure gradients occurring during continuous two-phase flow in small diameter vertical conduits. *Journal of Petroleum Technology (April 1965) 475; Tran., AIME, 234*.
- Hasan, R., & Kabir, S. (2005). A Simple Model for Annular Two-Phase Flow in Wellbores. *SPE Annual Technical Conference and Exhibition*. Dallas, Texas, U.S.A: Paper SPE 95523.
- Ivakhnenko, A. G. (1966). Group Method of Data Handling a Rival of the Method of Stochastic Approximation. *Soviet Automatic Control, 13*, 43-71.
- Jahanandish, I., Sakimifard, B., & Jalalifar, H. (2011). Predicting bottomhole pressure in vertical multiphase flowing wells using artificial neural networks. *Journal of Petroleum science and engineering*, 336-342.
- Mohaghegh, S., & Ameri, S. (1995). Artificial Neural Network As A Valuable Tool For Petroleum Engineers. *West Virginia University*. U.S.A. Telex: SPE 29220.
- Mohammadpoor, M., Shahbazi, K., Torabi, F., & Qazfani, A. (2010). A new methodology for prediction of bottomhole flowing pressure in vertical multiphase flow in Iranian oil fields using artificial neural networks (ANNs). *SPE latin american and caribbean petroleum engineering conference, 1-3, December 2010*. Lima, Peru.

- Moradi, B., Awang, M., & Shoushtari, M. A. (2011). Pressure Drop Prediction in Deep Gas Wells. *SPE Asia pacific oil & gas conference and exhibition, September 20-22, 2011*. Jakarta, Inonesia.
- Mukherjee, H., & Brill, J. P. (December 1985). Pressure DRop Correlations for Inclined Two-Phase Flow. *Journal of Energy Resources Technology*, 549-554.
- Orkiszewski, J. (1967). Predicting Two-Phase Pressure Drops in Vertical Pipe. *Journal of Petroleum Technology*, 829-838.
- Osman, E. A., & Abdel-Aal, R. E. (2002). Abductive Networks: A New Modeling Tool for the Oil and Gas Industry. *SPE Asia Pacific Oil and Gas Conference and Exhibition, 8-10 October 2002*. Melbourne, Australia: Research Institute, KFUPM, Dhahran, Saudi Arabia.
- Palisetti, R. V., & Heinze, L. (1998). Simplified Approach for Predicting Pressure Profiles in a Flowing Well. *SPE Permian Basin Oil and Gas REcovery Conference* , 23-26 March, 1998. Midland, Texas.
- Persad, S. (2005). Evaluation of Multiphae-Flow Correlations for Gas Wells Located Off the Trinidad Southeast Coast. *Latin American and Carribean Petroluem Engineering Conference*. Rio de Janeiro, Brazil, 20-23 June.
- Poettmann, F. H., & Carpenter, P. G. (n.d.). The Multiphase Flow of Gas, Oil, and Water Through Vertical Flow Strings with Application to the Design of Gas-lift Installations. 257-317.
- Pucknell, J., Mason, J., & Vervest, E. (1993). An Evaluation of Recent Mechanistic Models of Multiphase Flow for Predicting Pressure Drops in Oil and Gas Wells. *Paper SPE26682 , the 1993 offshore European Conference* , 7-10 September. Aberdeen.

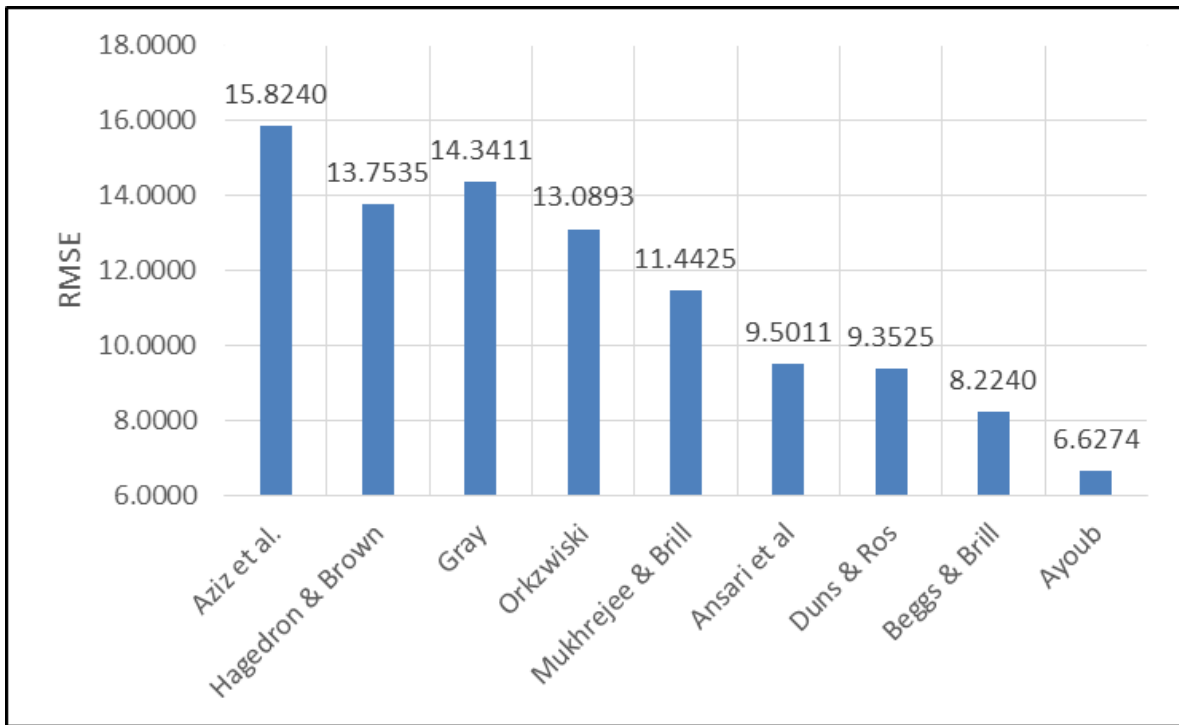
- Soni, S., Kelkar, m., & Perez, C. (2009). Pressure Drop Predictions in Tubing in The Presence of Surfactants. *SPE production and operations symposium*. Oklahoma, USA.
- Subsurface Controlled Subsurface Safety Valve Sizing Computer Program. (1978). In *Institute American Petroleum*. API Manual 14 BM, Second Ed., p. 38, API, Jan.
- Sukubo, I. A., & Igboanugo, A. C. (2011). Improved analytical model for predicting field production performance in vertical multiphase flow in pipes using MATLAB; A case study. *the Nigeria annual international conference and exhibition, 30 July - 3 August 2011*. Abuja, Nigeria.
- Takacs, G. (2001). Consideration on the Selection of an Optimum Vertical Multiphase Pressure drop Prediction Model for oil Wells. *SPE Production and Operation Symposium , 24-27 March 2001*. Oklahoma.
- Trick, M. D. (2003). Comparision of Correlations For Predicting Wellbore Pressure Losses in Gas-Condensate and Gas-Water Wells. *The Petroleum Society's Canadian International Petroleum Conference*. Calgary, Alberta: Neotechnology Consultants Ltd.
- Xiao, J. J., Shoham, O., & Brill, J. P. (1990). A Comperhensive Mechanistic Model for Two-Phase Flow in Pipelines. *65th Annuual technical Conference and Exhibition of the Society of Petroleum Engineering, September 23-26* (pp. 167-180). New Orleans, LA: University of Tulsa.
- Zafareh, F., Hill, A. D., & Podio, A. L. (1988). Flow Regimes In Vertical And Inclined Oil/Water Flow In Pipes. *63rd Annual technical conference and exhibition of the society of petroleum engineers, October 2-5*. Huoston, TX: U. of Texas.



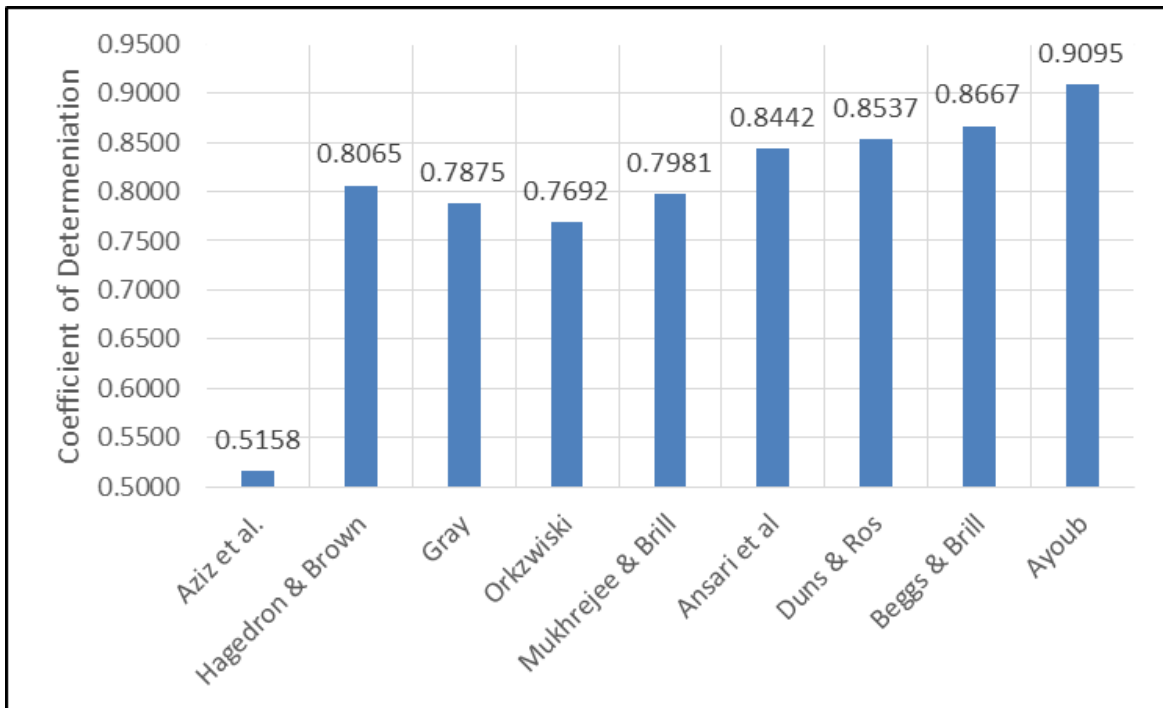
**Figure 1: The Absolute (AAPE) and Relative (APE) Percentage Error of All Models.**



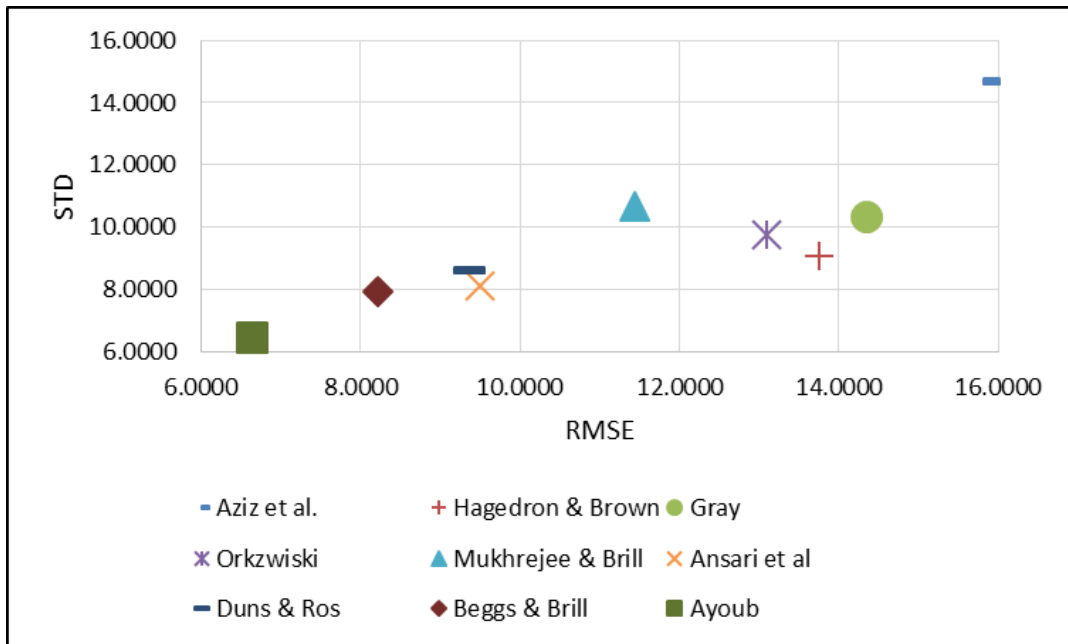
**Figure 2: Average Absolute Percentage Relative Error for All Models**



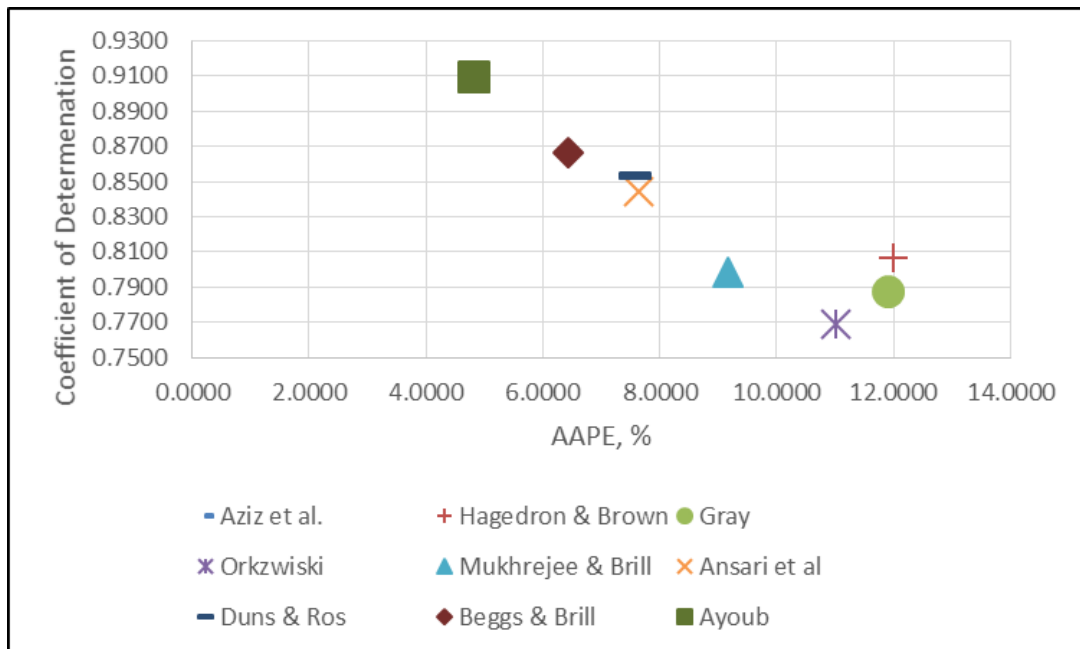
**Figure 3: Root Mean Square Error for All Models**



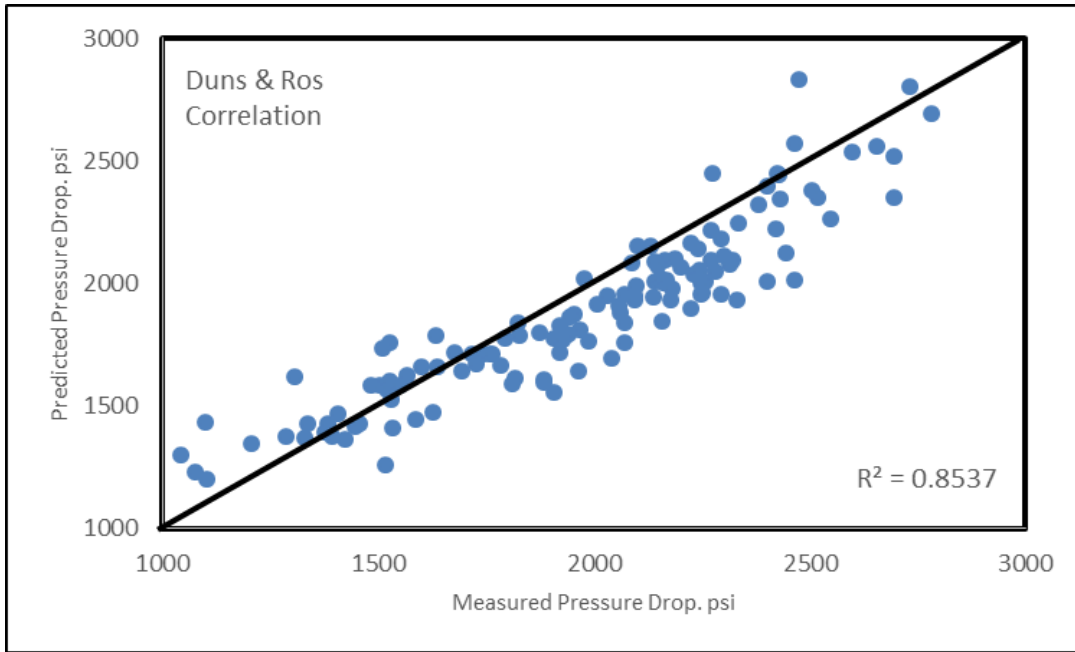
**Figure 4: Coefficient of Determination for All Models**



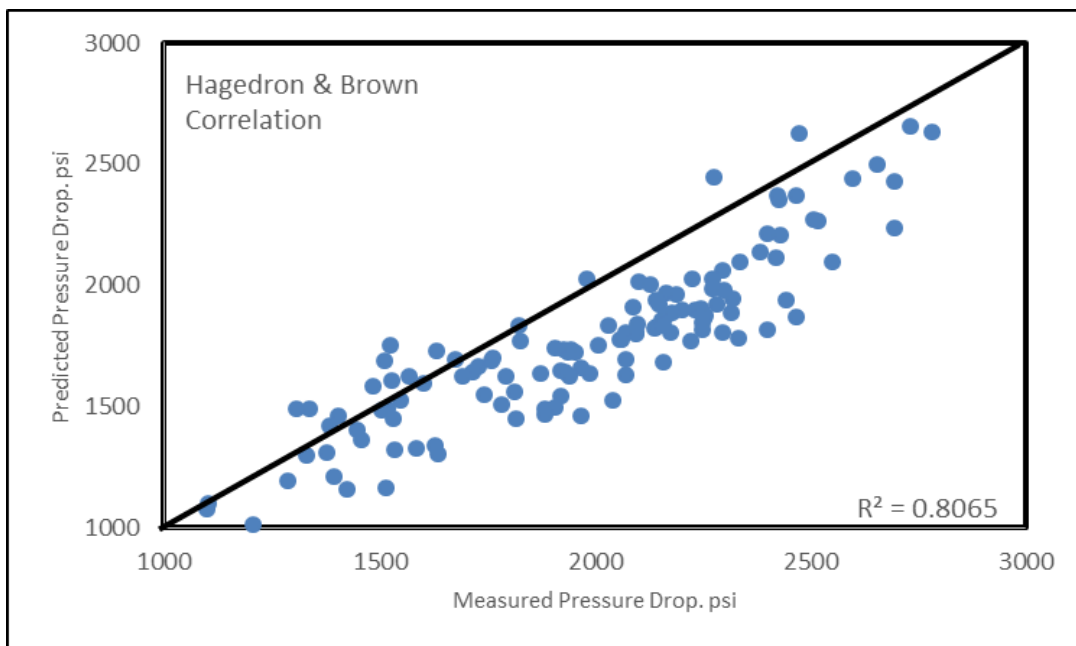
**Figure 5: Root Mean Square Error against Standard Deviation for All Models**



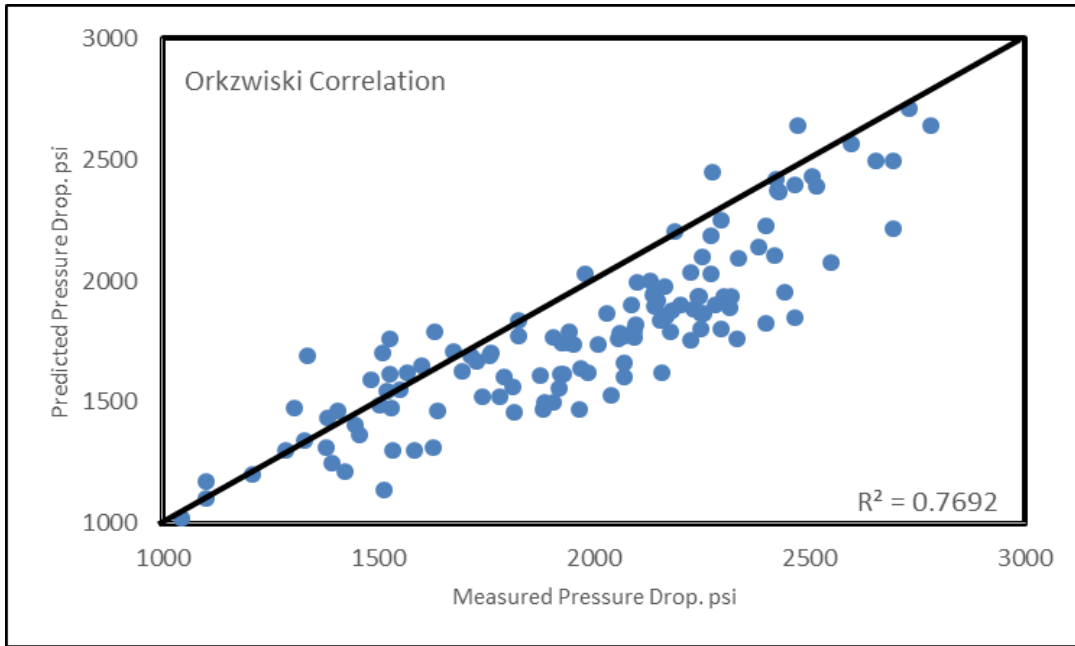
**Figure 6: Average Absolute Relative Error against the Coefficient of Determination for All Models**



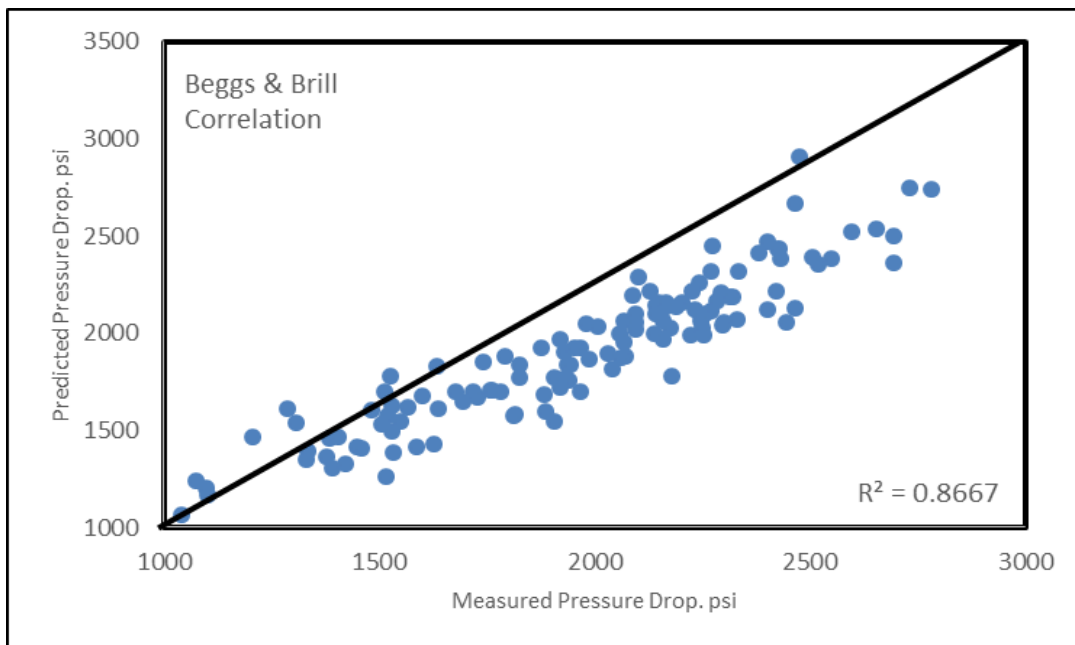
**Figure 7: Cross plot of pressure drop for Duns & Ros Correlation**



**Figure 8: Cross plot of pressure drop for Hagedron & Brown Correlation**

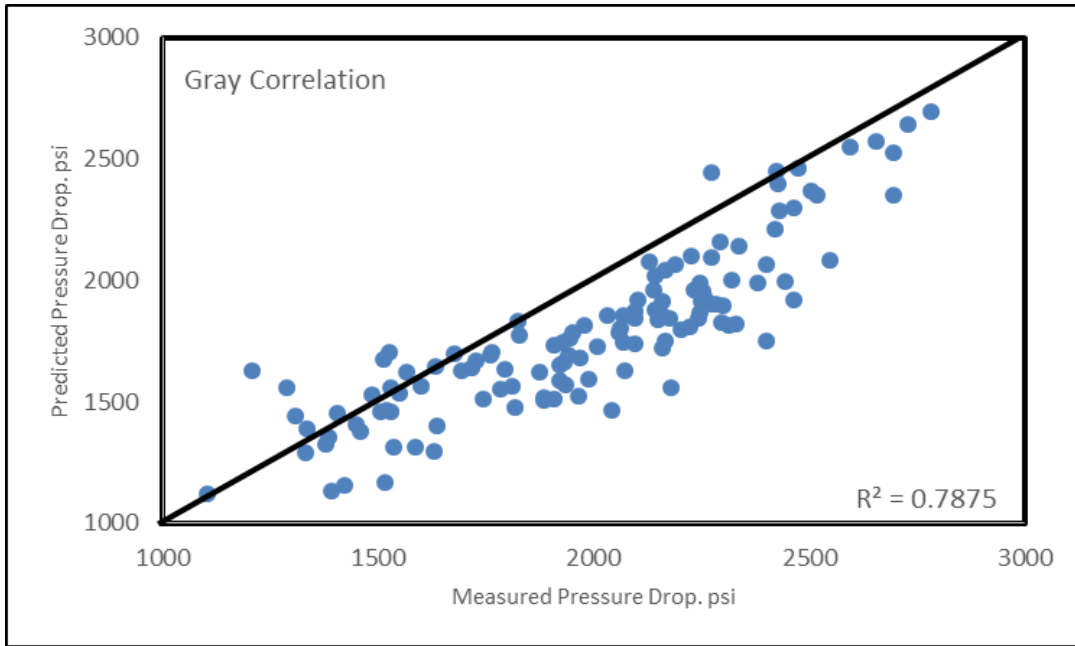


**Figure 9: Cross plot of pressure drop for Beggs Brill Correlation**

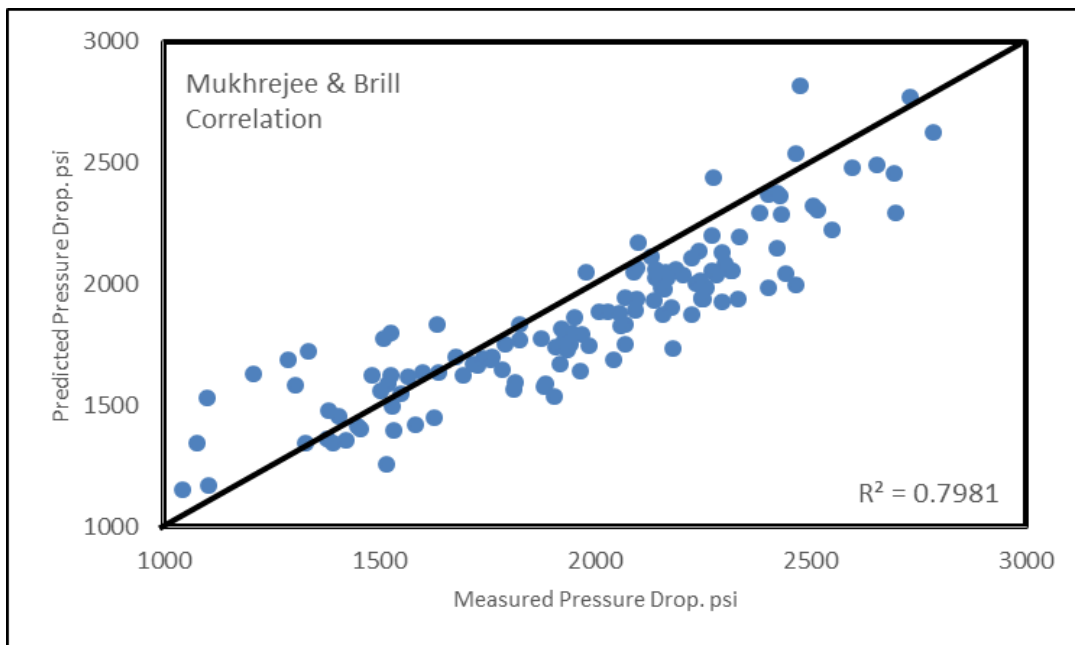


**Figure 10: Cross plot of pressure drop for Orkiszewski Correlation**

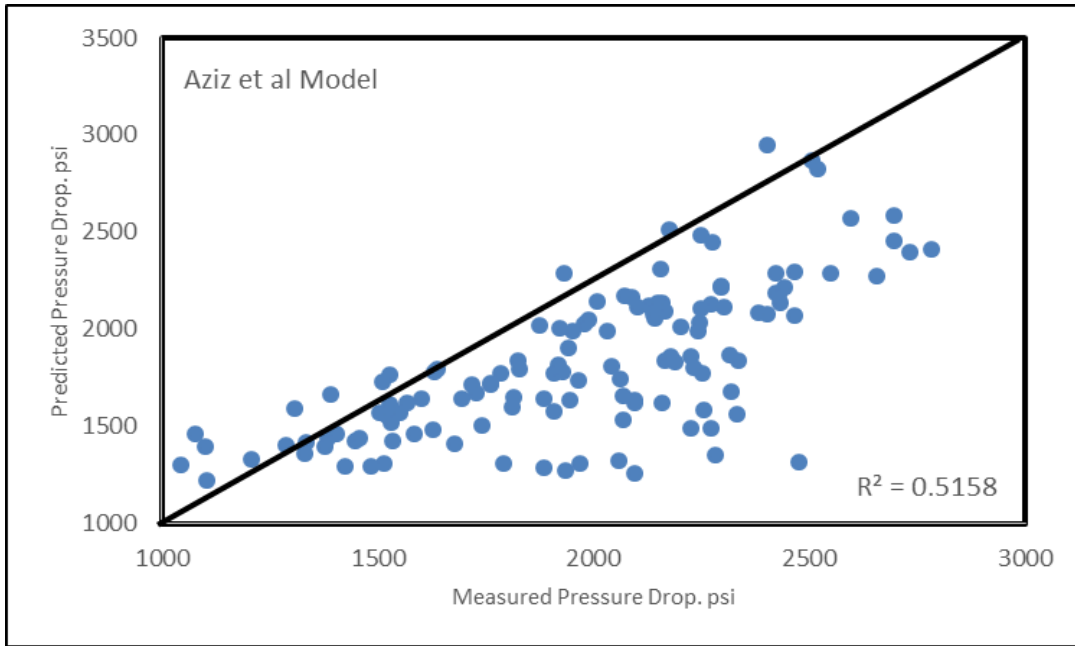




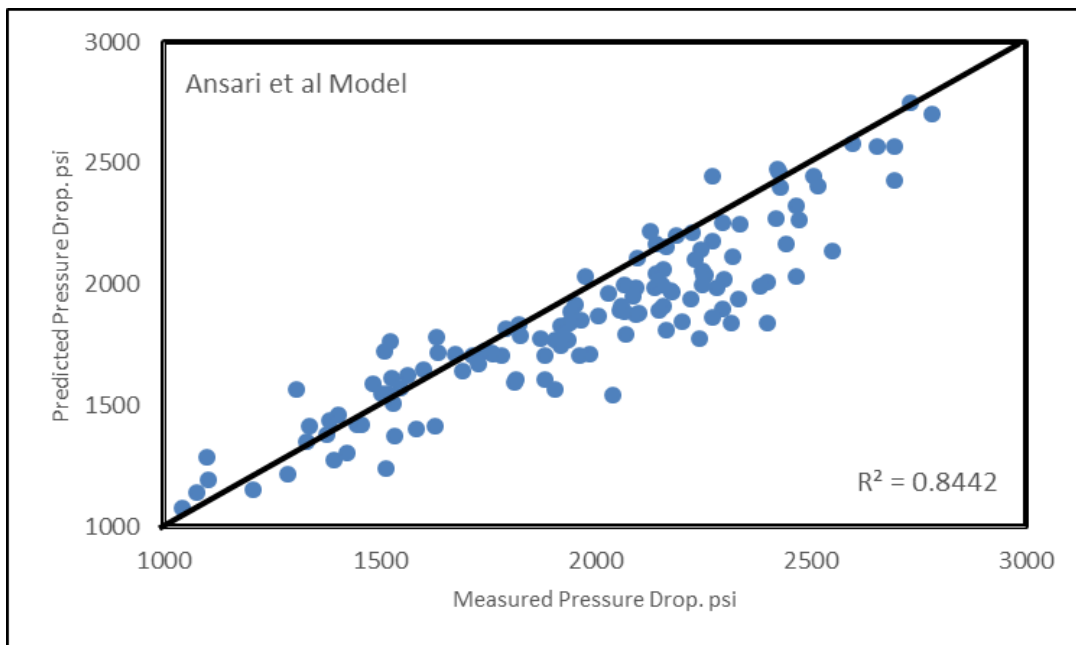
**Figure 11: Cross plot of pressure drop for Gray Correlation**



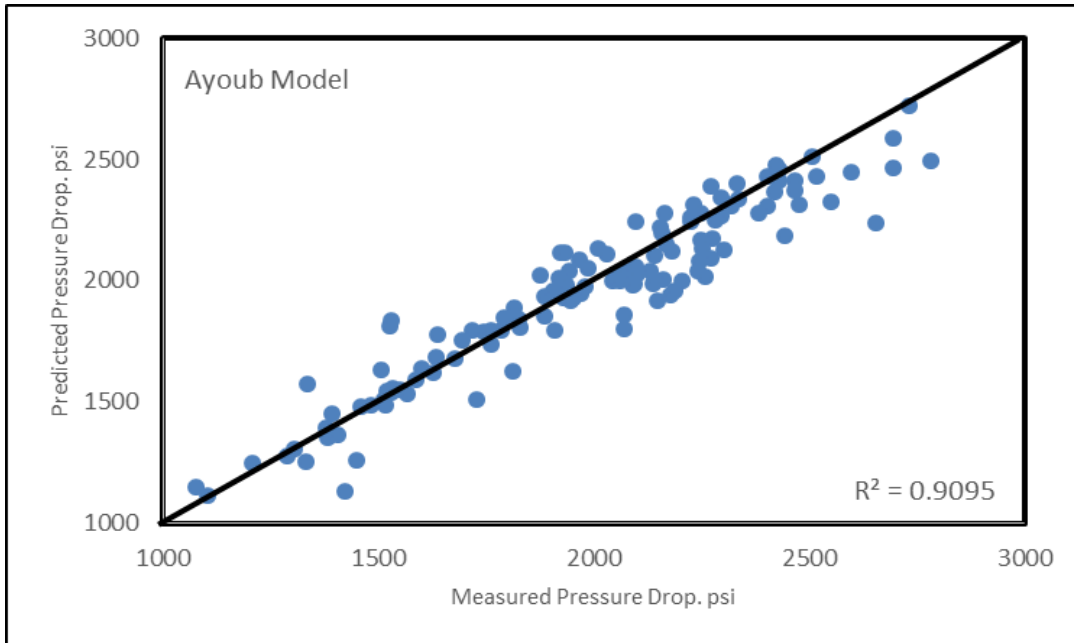
**Figure 12: Cross plot of pressure drop for Mukherjee & Brill Correlation**



**Figure 13: Cross plot of pressure drop for Aziz et al Model**



**Figure 14: Cross plot of pressure drop for Ansari et al Model**



**Figure 15: Cross plot of pressure drop for Ayoub Model**

### Appendix 1. Statistical Parameters

1. Average Absolut Percentage Relative Error:

$$E_a = \frac{1}{n} \sum_{i=1}^n |E_i|$$

2. Average Percentage Relative Error:

$$E_r = \frac{1}{n} \sum_{i=1}^n E_i$$

3. Maximum Absolute Relative Error:

$$E_{max} = \max_{i=1}^n |E_i|$$

4. Minimum Absolute Relative Error:

$$E_{min} = \min_{i=1}^n |E_i|$$

5. Root Mean Square Error:

$$RMSE = \left[ \frac{1}{n} \sum_{i=1}^n E_i^2 \right]^{0.5}$$

6. Coefficient of determination:

$$R^2 = \sqrt{1 - \frac{\sum_{i=1}^n [(\Delta P)_m - (\Delta P)_c]}{\sum_{i=1}^n [(\Delta P)_m - \overline{\Delta \Delta P}]}}$$

7. Standard Deviation:

$$STD = \sqrt{\left[ \left( \frac{1}{m - n - 1} \right) \left[ \sum_{i=1}^n \left\{ \frac{[(\Delta P)_m - (\Delta P)_c]}{(\Delta P)_m} \right\} * 100 \right]^2 \right]}$$

Where,  $E_i$  is the relative deviation of a calculated value from the measured value;

$$E_i = \left[ \frac{(\Delta P)_m - (\Delta P)_c}{\Delta P_m} \right] * 100\%, \quad i = 1, 2, 3, \dots, n$$

Where:

$(\Delta P)_m =$  Measured value pressure drop

$(\Delta P)_c =$  calculated value pressure drop

$$\overline{\Delta \Delta P} = \frac{1}{n} \sum_{i=1}^n [(\Delta P_m)]_i$$