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Natural Convective Coutte Flow in a Vertical Parallel Plate Microchannel

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Abstract. In the present paper, the exact analysis of steady state fully developed natural convective Couette flow in a vertical parallel plate microchannel is performed. Exact solutions are derived for the dimensionless velocity, temperature, volume flow rate, vertical heat flux and Nusselt number. The effects of Grashof number, wall-ambient temperature difference ratio and Knudsen number on the velocity, volume flow rate and Nusselt number have been discussed through graphs. The study revealed that the fluid velocity and volume flow rate increases with increasing Grashof number whereas the Nusselt number decreases with increasing Grashof number.

Introduction

The microfluidics become an important area of research in recent years due to its applications in microelctro mechanical systems (MEMS), biochemical cell reaction, biomedical sample injection, microelectric chip cooling, micropumps, microheat exchangers, sensors, and actuators etc. The modeling of fluid flow through microchannels depends on the Knudsen number, Kn, which is defined as the ratio of the molecular mean free path to the characteristic length scale of the physical system. For a gas flow with $10^{-3} < Kn < 10^{-1}$, the fluid domain can be treated as a continuum where the Navier-Stokes equations with slip boundary conditions are valid [1]. Chen and Weng [2] investigated the effects of rarefaction and fluid-wall interaction on steady fully developed natural convection flow in an open ended vertical parallel-plate microchannel with asymmetric wall temperature distributions. Their study revealed that the volume flow rate at microscale is higher than that at macroscale. Avic and Aydin [3] presented an analytical solution of steady fully developed mixed convection flow in a vertical microchannel with asymmetric wall temperatures. Avci and Aydin [4] was also presented an exact analytical results for fully developed mixed convection flow in a vertical parallel plate microchannel with asymmetric wall heat fluxes. Weng and Cheng [5] investigated the effects of variable thermal-physical properties on the steady state natural convection flow in a vertical parallel-plate microchannel at uniform wall temperatures. They observed that the variable properties effect increased the velocity slip and temperature jump for an air flow.

Recently, Buonomo and Manca [6] numerically investigated the transient natural convection in a parallel-plate vertical microchannel heated at uniform heat flux using finite difference method. They used the first-order model for slip velocity and temperature jump in microscale conditions. However, the steady state fully developed natural convective Couette flow in a vertical parallel-plate microchannel has not been studied in the literature even though it involves in biomedical sample injection, microactuators and microsensors etc. In the present paper the exact solution of fully developed natural convective Couette flow in a vertical parallel-plate microchannel has been studied under the assumptions of velocity slip and temperature jump conditions. The effects of Grashof number, wall-ambient temperature difference ratio and Knudsen number have been investigated on the fluid velocity, volume flow rate and Nusselt number.

Mathematical Analysis

Consider the steady laminar vicous incompressible natural convection flow in a vertical parallel-plate microchannel of a width b. The x – axis is taken along the plates in the vertical direction and the y – axis is normal to the plates. The cooler plate with temperature T_1 is located at $y = 0$ and the hotter plate with temperature $T_2\,(> T_1)$ is located at $y = b$ which is moving with uniform velocity u_s in its own plane in the upward direction. The channel ends are open to the ambient of temperature T_0 and density ρ_0. It is assumed that the fluid properties are constant, internal heat generation and viscous dissipative heat are negligible. Then under the usual Boussinesq approximation, the laminar boundary layer equations for hydrodynamically fully developed flow in the dimensionless form can be written as [2,3],

$$\frac{\partial^2 U}{\partial Y^2} + Gr\theta = 0 \tag{1}$$

$$RaU\frac{\partial \theta}{\partial X} = \frac{\partial^2 \theta}{\partial Y^2} \tag{2}$$

The corresponding dimensionless boundary conditions describing the slip velocity and temperature jump at fluid-wall interface are [1]:

$$U(0) = \beta_v \mathrm{Kn}\frac{\partial U(0)}{\partial Y} \tag{3}$$

$$U(1) = 1 - \beta_v \mathrm{Kn}\frac{\partial U(1)}{\partial Y} \tag{4}$$

$$\theta(0) = \xi + \beta_t \mathrm{Kn}\frac{\partial \theta(0)}{\partial Y} \tag{5}$$

$$\theta(1) = 1 - \beta_t \mathrm{Kn}\frac{\partial \theta(1)}{\partial Y} \tag{6}$$

The non-dimensional quantities introduced in Eqs (1) – (6) are defined as follows:

$$\left.\begin{aligned} &X = \frac{x}{b}, Y = \frac{y}{b}, U = \frac{u}{u_s}, \theta = \frac{T - T_0}{T_2 - T_0}, Gr = \frac{\rho_0 g\beta (T_2 - T_0) b^2}{\mu u_s}, Ra = \frac{\rho_0 c_p u_s b}{k}, \\ &\beta_v = \frac{2 - F_v}{F_v}, \beta_t = \frac{2 - F_t}{F_t}\frac{2\gamma_s}{\gamma_s + 1}\frac{1}{\mathrm{Pr}}, \gamma_s = \frac{c_p}{c_v}, \mathrm{Kn} = \frac{\lambda}{b}, \xi = \frac{T_1 - T_0}{T_2 - T_0}, In = \frac{\beta_t}{\beta_v}. \end{aligned}\right\} \tag{7}$$

Where b is channel width, c_p and c_v are specific heats at constant pressure and constant volume, respectively, F_t and F_v are the thermal and tangential momentum accommodation coefficients, respectively, g is gravitational acceleration, Gr is Grashof number, In is fluid-wall interaction parameter, k is thermal conductivity, Kn is Knudsen number, Pr is Prandtl number, Ra is Rayleigh number, T is fluid temperature, u and v are fluid velocity components in x and y

directions, respectively, u_s is hot plate velocity, U is dimensionless velocity component in x direction, X and Y are dimensionless rectangular coordinate system, β is thermal expansion coefficient, β_t and β_v are dimensionless variables, γ_s is ratio of specific heats, λ is molecular mean free path, μ is dynamic viscosity, θ is dimensionless temperature and ξ is wall-ambient temperature difference ratio.

For the fully developed temperature profile, $\frac{\partial \theta}{\partial X} = 0$ and equation (2) reduces to

$$\frac{\partial^2 \theta}{\partial Y^2} = 0 \tag{8}$$

Integrating equation (8) and applying the boundary conditions (5) and (6), we have

$$\theta(y) = A_1 y + A_2 \tag{9}$$

where

$$A_1 = \frac{1-\xi}{1+2\beta_v \mathrm{Kn} In} \tag{10}$$

$$A_2 = \frac{\xi + (\xi+1)\beta_v \mathrm{Kn} In}{(1+2\beta_v \mathrm{Kn} In)} \tag{11}$$

Integrating equation (1) and applying the boundary conditions (3) and (4), gives

$$U = B_2 + B_1 Y - \frac{GrA_2}{2} Y^2 - \frac{GrA_1}{6} Y^3 \tag{12}$$

where

$$B_1 = \frac{1}{(1+2\beta_v \mathrm{Kn})} + \frac{GrA_1}{6} \frac{(1+3\beta_v \mathrm{Kn})}{(1+2\beta_v \mathrm{Kn})} + \frac{GrA_2}{2} \tag{13}$$

$$B_2 = \frac{\beta_v \mathrm{Kn}}{(1+2\beta_v \mathrm{Kn})} + \frac{\beta_v \mathrm{Kn} Gr}{6} \left[A_1 \frac{(1+3\beta_v \mathrm{Kn})}{(1+2\beta_v \mathrm{Kn})} + 3A_2 \right] \tag{14}$$

The dimensionless volume flow rate, M, is

$$M = \frac{M'}{u_s b^2} = \int_0^1 U\, dY = B_2 + \frac{B_1}{2} - \frac{GrA_2}{6} - \frac{GrA_1}{24} \tag{15}$$

where M' is volume flow rate.

The total heat absorbed by the fluid in traversing the channel (or) the vertical heat flux is

$$Q = \frac{Q'}{\rho_0 c_p u_s (T_2 - T_0) b} = \int_0^1 U\theta\, dY$$
$$= \frac{1}{3} A_1 B_1 + A_2 B_2 + \frac{1}{2}\left(A_1 B_2 + A_2 B_1\right) - \frac{Gr}{6}\left(\frac{A_1^2}{5} + A_2^2 + A_1 A_2\right) \tag{16}$$

where Q' is vertical heat flux and Q is dimensionless vertical heat flux.

The dimensionless mean/bulk temperature, θ_m, is

$$\theta_m = \frac{T_m - T_0}{T_2 - T_0} = \frac{\int_0^1 U\theta\, dY}{\int_0^1 U\, dY} = \frac{Q}{M} \tag{17}$$

where T_m is mean/bulk temperature of the fluid.

The convective heat transfer coefficient, h, is given by

$$h = \frac{-k\left.\frac{\partial T}{\partial y}\right|_{y=b}}{T_m - T_2} = \frac{-\frac{k}{b}\left.\frac{\partial \theta}{\partial Y}\right|_{Y=1}}{(\theta_m - 1)} \tag{18}$$

and the rate of heat transfer is expressed in the form of Nusselt number, Nu, as

$$Nu = \frac{hb}{k} = \frac{MA_1}{(M - Q)} \tag{19}$$

Results and Discussion

The numerical values for dimensionless velocity, volume flow rate and Nusselt number are computed for the reference values of $F_v = 1, F_t = 1, \gamma_s = 1.4$ (i.e. $\beta_v = 1$ and $\beta_t = 1.667$) and $\Pr = 0.7$ [2]. The effect of thermal Grashof number (Gr) on fluid velocity is shown in Fig. 1. It is observed that an increase in Grashof number leads to an increase in the fluid velocity as well as the slip velocity at both walls. It means that the increase in thermal buoyancy force leads to large velocity slip in the microchannel. Also, it is observed that the maximum fluid velocity occurred near moving hot wall.

The effect of wall-ambient temperature difference ratio (ξ) on the fluid velocity is shown in Fig. 2. It is observed that the slip velocity at both walls and fluid velocity in the microchannel increases with the increase of temperature ratio. The effect of Knudsen number (Kn) on the velocity field is shown in Fig. 3. It is observed that the slip velocity increases with increasing the Knudsen number at the stationary wall while there is no slip velocity at the moving wall. That is, the slip induced by the rarefaction effect increases at the stationary wall whereas there is no slip at the moving wall. The variation of volume flow rate with Kn is shown in Fig. 4 at different values of *Gr*. It can be seen that the volume flow rate increases with increasing *Gr* and Kn except for $Gr = 0$. That is, volume flow rate increases with increasing buoyancy force and rarefaction.

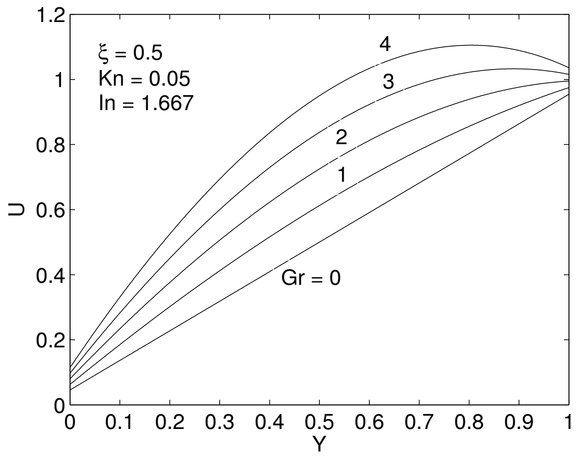

Fig. 1 Velocity distribution for different *Gr*

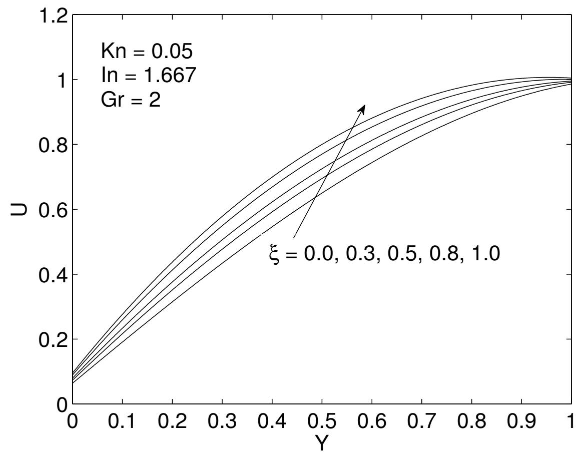

Fig. 2 Velocity distribution for different ξ

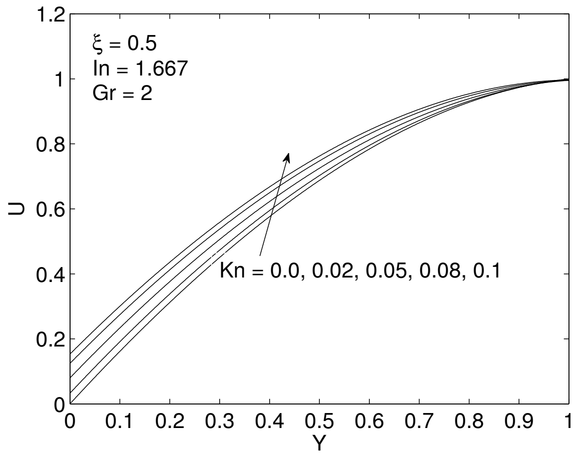

Fig. 3 Velocity distribution for different Kn

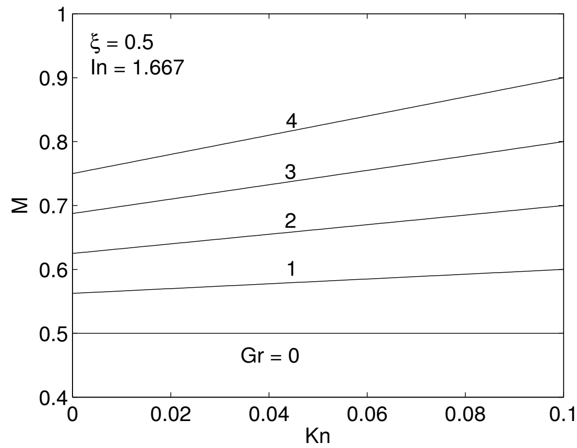

Fig. 4 Volume flow rate versus Kn at different Gr

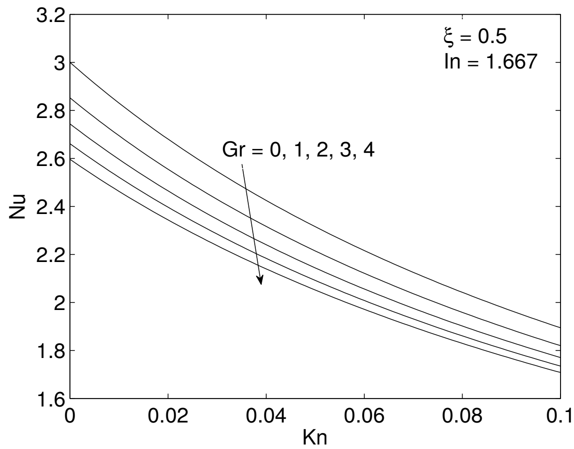

Fig. 5 Heat transfer rate versus Kn at different Gr

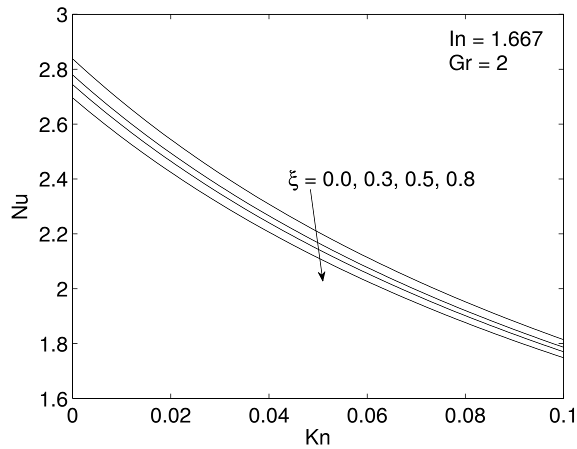

Fig. 6 Heat transfer rate versus Kn at different ξ

The variation of the Nusselt number with Kn is shown in Fig. 5 at different values of *Gr*. It is observed that the Nusselt number decreases with increasing Kn and *Gr*. That is, the rate of heat transfer decreases with increasing thermal buoyancy force as well as rarefaction. The effect of wall-ambient temperature difference ratio (ξ) on the Nusselt number is shown in Fig. 6. It can be seen that the Nusselt number decreases with increasing the wall-ambient temperature difference ratio.

Conclusions

The fully developed steady state natural convective Couette flow in a vertical parallel plate microchannel with constant but different wall temperature distributions has been studied analytically under the velocity-slip and temperature-jump boundary conditions. The influence of thermal Grashof number, wall-ambient temperature difference ratio and Knudsen number on the velocity field, volume flow rate and Nusselt number have been investigated. It is found that the slip velocity increases with increasing Knudsen number at the stationary wall while the slip velocity is absent at the moving wall. The Nusselt number is found to decrease as Knudsen number or wall-ambient temperature difference ratio or Grashof number increased.

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