Effect of the Spin-orbit Interaction on Partial Entangled Quantum Network

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***Abstract*—We use Dzyaloshiniskii-Moriya (DM) interaction to generate entangled network from partially entangled states in the presence of the spin-orbit coupling. The effect of the spin coupling on the entanglement between any two nodes of the network is investigated. It is shown that the entanglement decays as the coupling increases. For larger values of the spin coupling, the entanglement oscillates between upper and lower bounds. For initially entangled channels, the upper bound doesn't exceed its initial value, but for channels generated via indirect interaction, the entanglement reaches its maximum value.**

I. Introduction

Quantum Information Technology (QIT) promises faster, more secure means of data manipulation by making use of the quantum properties of matter [1]. One of the most important topics the QIT is generating entangled quantum networks. Quantum networks have implemented experimentally [2], [3], [4], [5] and theoretically [6], [7], [8], [9].

The Dzyaloshiniskii-Moriya (DM) interaction is a natural phenomena was discovered in 1960 by Moriya (Dzyaloshinskii-Moriya (DM) as an antisymmetric, anisotropic exchange coupling between two spins [10], [11]. It has been found that the DM interaction creates an strengthens entanglement among the particles which implies that the DM interaction plays an important role in the field of quantum networks [12]. The quantum correlation as a result of the DM interaction between two particles is investigated by many authors(see for examples [13], [14], [15]). The thermal entanglement between two qubits in the Heisenberg XYZ model and the effect of the DM interaction and its strength is discussed by Da-Chuang and Z.-Liang Cao [16]. The effect of the intrinsic decoherence on the teleportation of two qubits XYZ model is studied in the presence of DM interaction [17].

Metwally [6] introduced a theoretical protocol to generate multi-nodes quantum network by using maximum entangled states, where the terminals of each disconnected nodes are connected via DM interaction. The possibility of generating entangled network by using a class of partially entangled network is discussed by Abdel-Aty et. al, [7]. Therefore we are motivated to investigate the effect of the spin-orbit on the efficiency of the generated entangled network in the presence of DM interaction.

This paper is organized as follows: in section II the model and its evolution is introduced. The entanglement between the different nodes is quantifying for different values of the spin-orbits coupling and DM's strength in section III. Finally, our results are discussed in section V.

II. The Model

It is assumed that our aim is generating entangled network by using partially initially entangled states of Werner type [18], [7]. Consider a source generates pertial entangled state of the form

$$\rho_{ij} = \frac{1-F_w}{3} I_4 + \frac{4F_w - 1}{3} |\psi^-\rangle\langle\psi^-| \tag{1}$$

where $ij = 12, 34$, $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ is the singlet Bell state, and F_w is the maximal fraction corresponding to the Werner-state. The initial state of the total system is given by,

$$\rho_{1234}(0) = \rho_{12} \otimes \rho_{34} \tag{2}$$

The Hamiltonian which describe the evoluation of this system is given by,

$$\begin{aligned}\mathcal{H} \;=\;& J_x \sigma_x^{(i)} \sigma_x^{(j)} + J_y \sigma_y^{(i)} \sigma_y^{(j)} + J_z \sigma_z^{(i)} \sigma_z^{(j)} \\ & + D_x(\sigma_y^{(i)} \sigma_z^{(j)} - \sigma_z^{(i)} \sigma_y^{(j)})\end{aligned} \tag{3}$$

where i, j represent the nodes which will be connected via DM interaction which is considered in $x-$ axis with strength D_x , J_x, J_y, J_z are the real coupling coefficients in the x, y and z axise respectively, and $\sigma^{(i,j)} (i = x, y, z)$ are the Pauli matrices for the qubit i and (j). In our case i and j represent the second and third qubits respectively

The final density operator of the network is given by

$$\rho_{1234}(t) = \mathcal{U}(t)\rho_{1234}(0)\mathcal{U}^\dagger(t), \tag{4}$$

where,

$$\mathcal{U}(t) = e^{-i\mathcal{H}t}, \tag{5}$$

is a unitary operator defined by

$$\mathcal{U}^{(23)}(t) = \left[\cos(J_xt) - i\sin(J_xt)\sigma_x^{(2)}\sigma_x^{(3)}\right] \times\left[\cos(J_yt) - i\sin(J_yt)\sigma_y^{(2)}\sigma_y^{(3)}\right] \times\left[\cos(J_zt) - i\sin(J_zt)\sigma_z^{(2)}\sigma_z^{(3)}\right] \times\left[\cos^2(D_xt) + \sin^2(D_xt)\sigma_x^{(2)}\sigma_x^{(3)} - \frac{i}{2}\sin(2D_xt)(\sigma_z^{(2)}\sigma_y^{(3)} - \sigma_y^{(2)}\sigma_z^{(3)})\right] \quad (6)$$

In a matrix form, the unitary operator(6) can be written as

$$\mathcal{U}^{(23)}(t) = \begin{pmatrix} u_{ee,ee} & u_{ee,eg} & u_{ee,ge} & u_{ee,gg} \\ u_{eg,ee} & u_{eg,eg} & u_{eg,ge} & u_{eg,gg} \\ u_{ge,ee} & u_{ge,eg} & u_{ge,ge} & u_{ge,gg} \\ u_{gg,ee} & u_{gg,eg} & u_{gg,ge} & u_{gg,gg} \end{pmatrix}. \quad (7)$$

where,

$$u_{ge,eg} = [i\sin J_zt + \cos J_zt]\Big[\sin^2 D_xt (-\sin J_xt\sin J_yt + \cos J_xt\cos J_yt) - i\cos^2 D_xt\sin(t(J_x+J_y))\Big]$$

$$u_{ge,ge} = [i\sin J_zt + \cos J_zt]\Big[\cos^2 D_xt (-\sin J_xt\sin J_yt + \cos J_xt\cos J_yt) - i\sin^2 D_xt\sin(t(J_x+J_y))\Big]$$

$$u_{ge,gg} = \frac{1}{2}\sin D_xt(i\sin J_xt + \cos J_xt)(i\sin J_yt + \cos J_yt)(i\sin J_zt + \cos J_zt)$$

$$u_{gg,ee} = [\cos J_zt - i\sin J_zt]\Big[\sin^2 D_xt(\cos J_xt\cos J_yt + \sin J_xt\sin J_yt) - i\cos^2 D_xt\sin t(J_x - J_y)\Big],$$

$$u_{gg,ge} = -\frac{1}{2}\sin D_xt(i\sin J_xt + \cos J_xt)(i\sin J_yt - \cos J_yt)(i\sin J_zt - \cos J_zt)$$

$$u_{gg,gg} = [\cos J_zt - i\sin J_zt]\Big[\cos^2 D_xt(\cos J_xt\cos J_yt + \sin J_xt\sin J_yt) - i\sin^2 D_xt\sin t(J_x - J_y)\Big],$$

and the $u_{ge,gg} = -u_{ge,ee}, u_{gg,ge} = -u_{gg,eg} = u_{ee,eg} = -u_{ee,ge}, u_{ge,ge} = u_{eg,eg}, u_{ge,eg} = u_{eg,ge}, u_{gg,ee} = u_{ee,gg}$ and $u_{gg,gg} = u_{ee,ee}$.

Using Eq. (4) and Eq. (7), one gets the final entangled network between the four nodes. Since we are interested to quantify the degree of entanglement between the different nodes, one can obtain the required density operator between each to nodes by tracing out the other two nodes. For example the density operator between the first and the second nodes is given by $\rho_{12} = tr_{34}\{\rho_{1234}(t)\}$.

III. Results and Discussion

In this section, we quantify the entanglement between each two nodes. In practically, we consider the channels $\rho_{ij}, ij = 12, 13$ and 14. For this aim, we use Wootters's concurrent as a measure of entanglement [19] which is defined as,

$$\mathcal{C} = \max\{\sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}, 0\}, \quad (8)$$

where $\lambda_k, k = 1..4$ are the eigenvalues of the matrix $\rho_{ij}(\sigma_y^{(i)} \otimes \sigma_y^{(j)})\rho_{ij}^*(\sigma_y^{(i)} \otimes \sigma_y^{(j)})$.

The entanglement behavior (concurrence) of the entangled state between the nodes "1" and "2", ρ_{12} is described in Figs.(1-3) for different values of the coupling $J_i, i = x, y$ and z and the strength of DM is assumed to be fixed, $D_x = 0.2$. Fig.(1) describes the evolution of the concurrence $\mathcal{C}$ in the presence of zero coupling or only one non-zero coupling. It is clear that for $j_x = J_y = J_z$, the concurrence decays gradually to reach its minimum bounds ($\mathcal{C} = 0.4$), then increases to reach its maximum bounds, which don't exceed the initial bounds. This show that the decay due to the interaction of the second nod with the third nodes, so there are some correlation are lost. However, when only one non-zero coupling is switched on, the upper and lower bounds depend on this coupling. The behavior shows that, the minimum bounds of $\mathcal{C}$ for $J_x \neq 0$ are always larger than that depicted for $J_y \neq 0$ or $J_z \neq 0$. On the other hand, for $J_y \neq 0$ or $J_z \neq$, the concurrence vanishes completely i.e., $\mathcal{C} = 0$ as the scaled time increases and the upper bounds don't exceed the initial value [6], [7]. In Fig.3, we investigate the behavior of the concurrence where we consider $J_x = J_y = J_z \neq 0$. It is clear that for small values of the coupling parameters. i.e., $J_x = J_y = J_z = 0.1$, the concurrence $\mathcal{C}$ decays gradually and vanishes once for $t \in [7, 8]$. However the upper bounds don't exceed the initial value namely at $t = 0$. For larger values of $J_i, i = x, y, z$, the concurrence decays faster but the lower bounds are non-zero and the upper bounds are slightly increase.

Fig.(2) describes the behavior of the concurrence for the entangled state ρ_{12}, where two two-non-zero coupling are

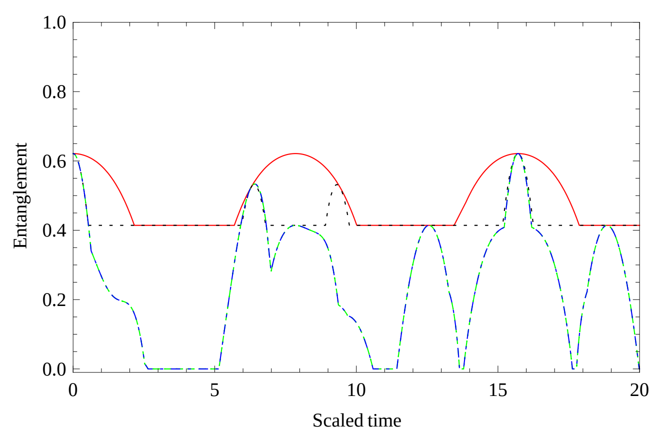

Fig. 1. The dynamics of the entanglement between node 1 and node 2 $\mathcal{C}_{12}$, where the red line with $J_x = J_y = J_z = 0$ (without the effect of spin) and black dot line represent the entanglement with $J_x = 0.5$ and $J_y = J_z = 0$ green line when $J_y = 0.5$ and $J_x = J_z = 0$ and blue dot dash line the 12 entanglement with $J_x = J_y = 0$ and $J_z = 0$ with $D_x = 0.2$

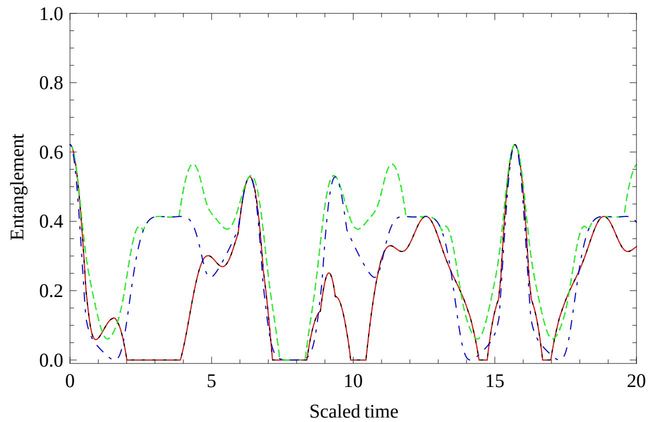

Fig. 2. The same with fig 1 but where the red line with $J_x = J_y = 0.5$ and $J_z = 0$ and black dot line represent the entanglement with $J_x = J_y = 0.5$ and $J_z = 0$ green line when $J_x = 0$ and $J_y = J_z = 0.5$ and blue dot dash line the 12 entanglement with $J_x = J_y = J_z = 0.5$ and with $D_x = 0.2$

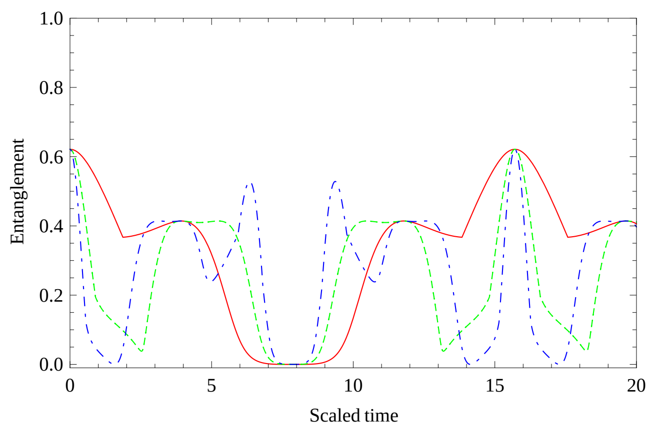

Fig. 3. The dynamics of the entanglement between node 1 and node 2 $\mathcal{C}_{12}$, where the red line with $J_x = J_y = J_z = 0.1$ and dash green line when $J_x = J_y = J_z = 0.3$ and blue dot dash line with $J_x = J_y = J_z = 0.5$ with $D_x = 0.2$

considered. The general behavior is the same as that depicted in Fig.(1), but the number of oscillations between the upper and lower bounds increases. However, if we compare the solid curves in Figs.(1&2), we can see that the presences of the coupling cause a faster decay of the concurrence.

Figs.(4-6), describe the behavior of the concurrence for the entangled state which is generated between the nodes "1" and "3" via direct interaction. Fig.(4) describes the behavior of $\mathcal{C}$ for onle one non-zero coupling is considered, where we use the same values of the coupling and DM's strength as considered in Fig.(1). Sice the two nodes are initially disentangled, then at $t = 0$, the concurrence $\mathcal{C} = 0$. However as soon as the interaction is switched on an entangled state is generated between the first and the third nodes and consequently the concurrence increases to reach its upper bounds ($\mathcal{C} = 0.4$). However for farther increasing t, the concurrence decays to vanishes completely. This behavior is periodically repeated.

The dynamics of the concurrence, $\mathcal{C}$ for the channel ρ_{13} when two non-zero couplings are considered is displayed in Fig.(5). The values of $J_i, i = x, y, z$ are the same for Fig.(2). This figures shows that, the concurrence oscillates between

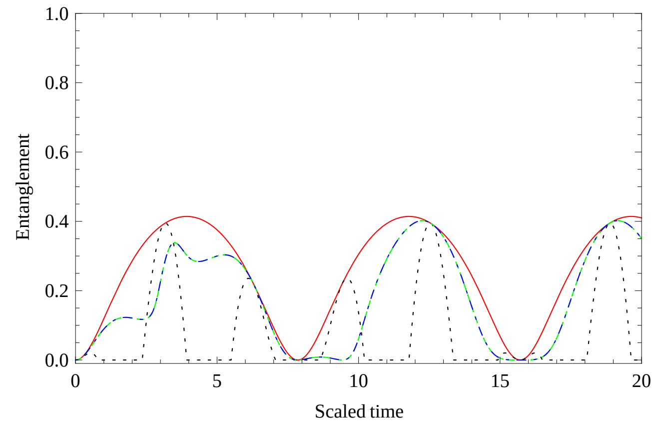

Fig. 4. The same with Fig.(1) but for the channel ρ_{13} which is generated between the first and third nodes via direct interaction.

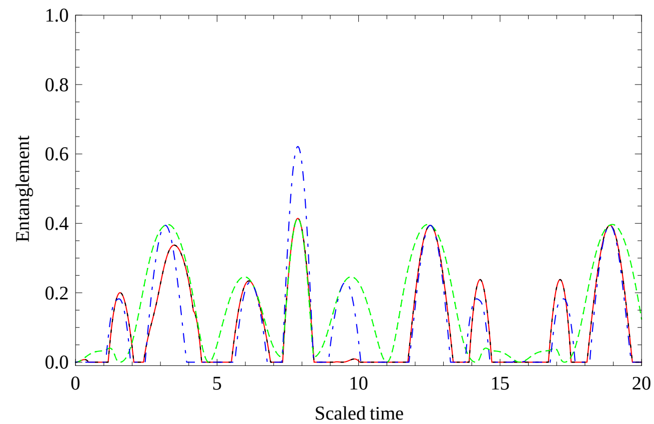

Fig. 5. The same with Fig. 2 but for the channel ρ_{13}

its lower and upper bounds very fast. The phenomena of the sudden-death and birth appear of the entanglement are appear clearly.

Fig.(6) describes the behavior of $\mathcal{C}$ for the state ρ_{13} when $J_x = J_y = J_z \neq 0$, where $J_i, i = x, y, z$ are given the same values in Fig.(3). It is clear the general behavior is similar to that depicted in Fig.(5), but the number of oscillations increases as the values of the coupling increase. However, the upper bounds are slightly larger for larger J_i.

Finally, we investigate the entanglement behavior for the state ρ_{14} as in Figs. (7-9), which is generated via indirect interaction. It is clear that the behavior of $\mathcal{C}$ is similar to that displays for ρ_{13}. However for the state ρ_{14}, the upper bounds are much larger and reach to their maximum values, i.e, $\mathcal{C} = 1$. The number of oscillations of the concurrences as the spin-orbit coupling increases. On the other hand, these oscillations increases if all the couplings have non-zero values.

IV. Conclusion

We discussed the effect of the spin-orbit coupling on the entanglement between different nodes of quantum network. In general, the entanglement decays for non-zero values of the coupling. The phenomena of the sudden-death and sudden-birth appear for larger values of the coupling.

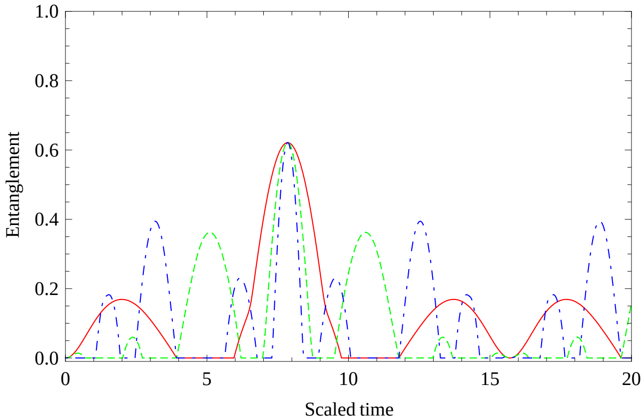

Fig. 6. The dynamics of the entanglement between node 1 and node 3 $\mathcal{C}_{13}$, where the red line with $J_x = J_y = J_z = 0.1$ and dash green line when $J_x = J_y = J_z = 0.3$ and blue dot dash line with $J_x = J_y = J_z = 0.5$ with $D_x = 0.2$

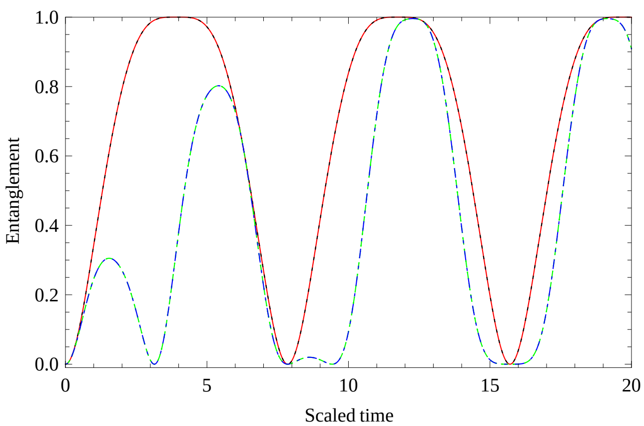

Fig. 7. The same with Fig. 1 but for the channel ρ_{14}

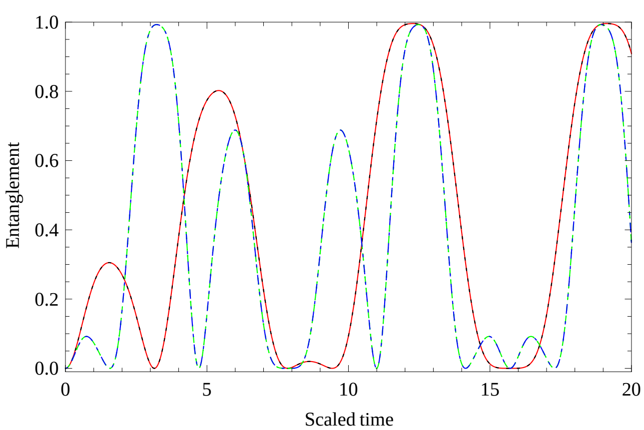

Fig. 8. The same with Fig. 2 but for the channel ρ_{14}

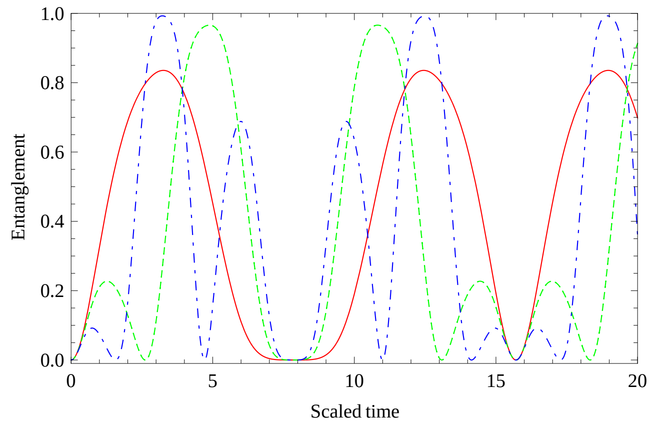

Fig. 9. The dynamics of the entanglement between node 1 and node 4 $\mathcal{C}_{14}$, where the red line with $J_x = J_y = J_z = 0.1$ and dash green line when $J_x = J_y = J_z = 0.3$ and blue dot dash line with $J_x = J_y = J_z = 0.5$ with $D_x = 0.2$

It is shown that, for initially entangled channel, the coupling constant has no effect on the upper bound of entanglement. However, for non-zero couplings, the lower bounds of entanglement do nor vanish. The number of oscillations is increased as the the coupling is increased. For entangled channels which generated via direct or indirect interaction, the concurrence and the number of oscillations are increased as coupling is increased.

Finally, it is shown that, the generated entangled channel between any two nodes via indirect interaction has a large degree of entanglement. The upper bounds exceed the initial entangled state. Therefor, from less entangled state one can generated maximum entangled states by controlling the spin-orbit coupling. This means that, one can use the terminals of the generate entangled network to perform quantum information tasks with high efficiency.

References

[1] Isaac L. Chuang Michael A. Nielsen. Quantum computation and quantum information. In *Cambridge University Press*, 2000.

[2] Y. C. Elliott. Building the quantum network. *New J. Phys*, 4(3):46 –53, jul/sep 2002.

[3] A. Mink B. J. Hershman A. Nakassis X. Tang R. Lu D. H. Su C. W. Clark C. J. Williams E. W. Hagley J. Wen J. Bienfang, A. J. Gross. Quantum key distribution with 1.25 gbps clock synchronization. *Optics Express*, 12(9):2011–2016, May 2004.

[4] F. Buntschu B. Clausen N. Felber N. Gisin L. Henzen P. Junod G. Litzistorf P. Monbaron L. Monat J-B. Page D. Perroud G. Ribordy A. Rochas S. Robyr J Tavares R. Thew P. Trinkler S. Ventura R. Voirol N. Walenta D. Stucki, M. Legr and H. Zbinden. Long-term performance of the swissquantum quantum key distribution network in a field environment. *New J. Phys*, 13:123001, 2011.

[5] H. Ishizuka W. Klaus K. Wakui M. Takeoka A. Tanaka K. Yoshino Y. Nambu S. Takahashi A. Tajima A. Tomita T. Domeki T. Hasegawa Y. Sakai H. Kobayashi T. Asai K. Shimizu T. Tokura T. Tsurumaru M. Matsui T. Honjo K.Tamaki H. Takesue Y. Tokura J. F. Dynes A. R. Dixon A. W. Sharpe Z. L. Yuan A. J. Shields S. Uchikoga M. Legre S. Robyr P. Trinkler L. Monat J.-B. Page G. Ribordy A. Poppe A. Allacher O. Maurhart T. Langer M. Peev A. Zeilinger. M. Sasaki, M. Fujiwara. Field test of quantum key distribution in the tokyo qkd network. *Optics Express*, 19(11):10387–10409, M 2011.

[6] M. Mtwallay. Entangled network and quantum communications. *Phys. Lett. A*, 375(3):4268 –53, jul/sep 2011.

[7] Abdel-Haleem Abdel-Aty, LeeYen Cheong, Nordin Zakaria, and Nasser Metwally. Quantum network via partial entangled state. *APC Conf. Proc. Uinversiti Teknologi Petronas*, page Submitted, 2013.

[8] Abdel-Haleem Abdel-Aty, LeeYen Cheong, Nordin Zakaria, and Nasser Metwally. Entanglement and teleportation via partial entangled-state quantum network. *In preparation*, 2013.

[9] Darwish M. Obada A.-S.F. El-Barakaty, A. Purity loss for a cooper pair box interacting dispersively with a nonclassical field under phase damping. *Appl. Math. Inf. Sci*, 5:122, 2011.

[10] Tôru Moriya. New mechanism of anisotropic superexchange interaction. *Phys. Rev. Lett*, 4(5):228, Mar 1960.

[11] M. Friesen S. Chutia and R. Joynt. Detection and measurement of the dzyaloshinskii-moriya interaction in double quantum dot systems. *Phys. Rev. B*, 73(24):241304, Jun 2006.

[12] Guo-Feng Zhang. Thermal entanglement and teleportation in a two-qubit heisenberg chain with dzyaloshinski-moriya anisotropic antisymmetric interaction. *Phys. Rev. A*, 75:034304, Mar 2007.

[13] Sha-Sha Li, Ting-Qi Ren, Xiang-Mu Kong, and Kai Liu. Thermal entanglement in the heisenberg {XXZ} model with dzyaloshinskiimoriya interaction. *Physica A: Statistical Mechanics and its Applications*, 391(12):35 – 41, 2012.

[14] Ye Yeo. Teleportation via thermally entangled states of a two-qubit heisenberg *XX* chain. *Phys. Rev. A*, 66:062312, Dec 2002.

[15] Ming-Yong Ye, Wei Jiang, Ping-Xing Chen, Yong-Sheng Zhang, Zheng-Wei Zhou, and Guang-Can Guo. Local distinguishability of orthogonal quantum states and generators of su(N). *Phys. Rev. A*, 76:032329, Sep 2007.

[16] Da-Chuang Li and Zhuo-Liang Cao. Thermal entanglement in the anisotropic heisenberg {XYZ} model with different inhomogeneous magnetic fields. *Optics Communications*, 282(6):1226 – 1230, 2009.

[17] Zhenghong He, Zuhong Xiong, and Yanli Zhang. Influence of intrinsic decoherence on quantum teleportation via two-qubit heisenberg {XYZ} chain. *Physics Letters A*, 354(12):79 – 83, 2006.

[18] A. S. Majumdar S. Adhikari and B. Ghosh N. Nayak, S. Roy. Teleportation via maximally and non-maximally entangled mixed states. *Quant. Info. Comp.*, 10:0398, Mar 2010.

[19] Scott Hill and William K. Wootters. Entanglement of a pair of quantum bits. *Phys. Rev. Lett.*, 78:5022–5025, Jun 1997.