Automated Model Generation of Analog Circuits Through Modified Trajectory Piecewise Linear Approach With ChebyShev Newton Interpolating Polynomials

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Abstract—**In this paper we propose an efficient scalability approach for the trajectory piecewise linear (TPWL) macromodels through the utilization of Chebyshev interpolating polynomials in each piecewise region. The scalability achieved is in two dimensions (2D) that mainly improve the local approximation properties of TPWL macromodels. Horizontal scalability is achieved by decreasing the number of linearization points along the trajectory; vertical scalability is obtained by extending the range of macromodel to predict the response of a nonlinear system for inputs far from training trajectory. In this way more efficient macromodels are obtained in terms of simulation speed up of complex nonlinear systems. We provide the implementation details and illustrate the 2D scalability concept with an example using nonlinear transmission line.**

Keywords-Trajectory; Chebyshev polynomial; Taylor polynomial; State space (SS); Model order reduction (MOR)

I. INTRODUCTION

Model Order Reduction (MOR) techniques used for macromodeling of nonlinear systems [1–3],[4–8] rely on the piecewise linearization (PWL) of original system response. Linearization is performed by expanding nonlinear function into its Taylor polynomials. Trajectory methods like [4],[9] use linear and quadratic Taylor polynomials in their piecewise regions. However, due to poor convergence properties of Taylor polynomials, these approaches cannot guarantee the correct response for the inputs that are not included in training or have greater magnitudes than training inputs.

It is intuitive to use efficient orthogonal polynomials like Chebyshev polynomial that can interpolate non-reachable states better than Taylor interpolating polynomials. In this work we propose 2D scalability of TPWL macromodels by combining Chebyshev-Newton polynomials with nonlinear state space structure.

Previous attempts (e.g., [10]) use Chebyshev polynomials standalone to approximate nonlinear functions in electronic circuits. To the best of our knowledge this is the first implementation of Chebyshev polynomials in SS model structure to predict the response of nonlinear electronic circuits and system.

II. BACKGROUND

A. Macromodeling of Nonlinear Systems

A nonlinear dynamical system can be described by a SS approach [4] shown in equation (1).

$$\begin{cases} \dfrac{dg\,(x(t))}{dt} = f\,(x(t)) + B\,(x(t))u\,(t) \\ y(t) = C^T x(t) \end{cases} \tag{1}$$

where $x(t) \in R^N$ is state vector at time t representing the unknown node voltages and branch currents; $f : R^N \rightarrow R^N$ and $g : R^N \rightarrow R^N$ are nonlinear vector valued functions representing charge/flux and resistive terms in the circuit respectively; B is a SS-dependent $N \times M$ input matrix that can be considered as constant if its value does not change with time; u: $R \rightarrow R^M$ is the input signal; C is a $N \times K$ output matrix and $y : R \rightarrow R^K$ is the output signal.

Several MOR techniques have been developed to model strong and weak nonlinear circuits and systems using the SS approach in equation (1) [4–8]. Two of the most popular techniques for modeling strong nonlinear systems using SS are Trajectory Piecewise Linear (TPWL) [4] and Piecewise Polynomial (PWP) [9].

B. Chebyshev Polynomials

Chebyshev polynomials of first kind are given in equation (2) [12], where x' are the nodes of these polynomials; N is the order of polynomial.

$$T_{N+1}(x') = \cos((N+1)\cos^{-1} x) \tag{2}$$

The nodes of Chebyshev polynomials in equation (2) are calculated using equation (3) as long as trajectory amplitude lies in the interval [-1,1].

$$x'_k = \cos\frac{2N+1-2k}{2(N+1)}\pi \qquad k=0,1\ldots,N \tag{3}$$

For an arbitrary interval [a,b], x needs to be normalized and nodes are calculated according to equation (4).

$$x'_k = \frac{b-a}{2}\cos\frac{2N+1-2k}{2(N+1)}\pi + \frac{a+b}{2} \qquad k=0,1\ldots,N \tag{4}$$

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To approximate a function *f(x)*, discrete orthogonality of Chebyshev polynomials can be exploited to define the truncated Chebyshev series shown in equation (5) [12].

$$f(x) \cong c_N(x) = \sum_{m=0}^{N} d_m T_m(x') \bigg|_{x'=\frac{2}{b-a}(x-\frac{a+b}{2})} \quad (5)$$

where

$$d_m = \frac{2}{N+1}\sum_{m=0}^{N} f(x_k)\cos\frac{m(2N+1-2k)}{2(N+1)}\pi \quad \textit{for m=1,2...,N}$$

To generate interpolating polynomials that can be practically simulated with SS of form (1), a combination of Netwon polynomials with Chebyshev knots [11] can be used. This implies that the knots of interpolating polynomials are generated using chebyshev knot formulae given in equation (5) or (6) and then generate the polynomial using Newton divided difference method. Resulting polynomial for approximating the function *f(x)* is given by equation (6), where $c_1, c_2, c_3..c_n$ are the polynomial coefficients.

$$f(x) \approx c_1 x^{n-1} + c_2 x^{n-2} + c_3 x^{n-3} + \ldots + c_n^1 \quad (6)$$

III. Chebyshev Polynomials For 2D Scalability Of Local Models

A. 2D Scalability of the Trajectory Macromodels

It has been shown in section II that TPWL utilizes pure linear model in each linearized region, whereas PWP uses higher order Taylor polynomials to improve local approximation in each piecewise region. However, due to limited region of convergence of Taylor polynomials, these polynomials are unable to interpolate the non-reachable states that are not excited during model training. Chebyshev polynomials guarantee convergence inside an interval [a,b], as long as no singularities lies within the interval [12]. This means that higher order Chebyshev polynomials can be employed confidently in each region to interpolate non-reachable states during simulation. Advantages of using Chebyshev polynomials in SS are twofold. Primarily, due to guaranteed convergence it allows to use higher order polynomials in each region. Secondly, it reduces the overall number of linearization points used for model generation.

Consider a function $f(x) = \frac{x^2 \exp^{-\frac{x^2}{2}}}{\sqrt{2\pi}} + x\sin(2\pi x)$, which is evaluated at 100 points of x = -2.5:0.05:2.5. All points are used for curve fitting. Taylor series expansions are performed around the origin. One can observe from figure1 that as long as input x is small within the range of (-1,1), Taylor polynomial fit is better (at the expense of high polynomial orders).

Outside the region, Taylor polynomials start diverging from solution and even increasing order does not show significant difference. On the other hand, single Chebyshev polynomial (of relatively high order of 20) is able to fit the complete curve.

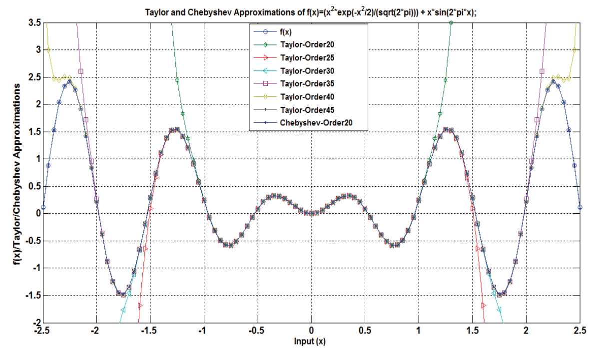

Fig. 1. Higher degree Chebyshev and Taylor polynomial approximations

This implies that for large input swing, Chebyshev models can cover the large SS regions as compared to Taylor polynomials, where multiple linearization points with many polynomials are used to cover large inputs. This also provides the intuition that total number of linearization points can be *reduced* using Chebyshev polynomials, thus enabling fast simulation of macromodels.

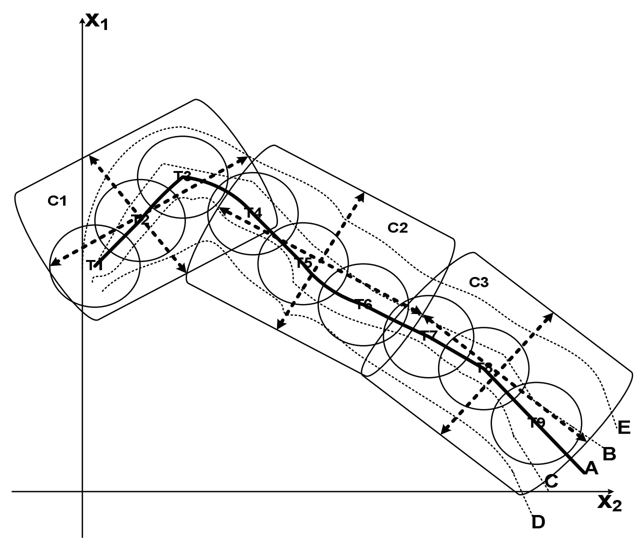

Fig. 2. Generation of linearized models using Taylor and Chebyshev polynomials along a trajectory of nonlinear system in a 2D SS

Consider an example illustrated in figure 2, Nine linearization points are generated along a training trajectory *A* using Taylor polynomials. It can be seen that as long as input is closer to linearization points, trajectories (*B* and *C*) fall inside the regions of linearization points. Once input is far from training input, other trajectories (D and E) are distant from *A*. However, large rectangles representing Chebyshev linearization regions are able to cover all trajectories far from training trajectory *A*. It implies that Chebyshev polynomials can improve the scalability of

TPWL models and increase the scope of a macromodel in 2D: horizontally the number of linearization points is reduced; vertically the span is increased to cover more trajectories far from training trajectory (2D extensions can be observed from dashed lines in figure 2).

B. Representation of nonlinear function with Chebyshev polynomials

Without lost of generality, state equation (1) can be expressed as

$$E\frac{d}{dt}x = f(x) + Bu(t) \qquad (7)$$

where function *g(x)* is considered to be linear and is replaced with a constant *E*. Similarly, *B* is considered to be time invariant.

The nonlinear function in equation (7) is *f(x)* that can be approximated using Chebyshev interpolating polynomial. Replacing *f(x)* in equation (7) with n^{th} order Chebyshev interpolating polynomial, resulting state equation has the form shown in equation (8).

$$E\frac{d}{dt}x = \{c_1x^{n-1} + c_2x^{n-2} + c_3x^{n-3} + ... + c_n^1\} + Bu(t) \qquad (8)$$

where $c_1, c_2, ..., c_n$ are the polynomial coefficients computed using Newton divided difference method [11].

IV. Macromodel Generation Using Chebyshev Polynomials

In this section we provide steps required to obtain macromodels using Chebyshev interpolating polynomials.

A. Macromodel Training Input

In order to generate accurate macromodels, the macromodel needs to be trained with the input that can excite all possible states of SS. A good training input should be strong enough that it forces the SS to its upper bounds [9].

Typically for training, trajectories are generated by combining signals of different amplitudes, frequencies and phases. In this study, we use single training data by superimposing a 500mV, 10*us* PRBS signal on 120Hz, ±2 V sinusoidal. A High frequency PRBS signal is expected to excite all possible states of SS and high input values force SS to its maximum limits.

B. Macromodel Generation

The aim of using Chebyshev polynomials in SS is to generate *minimum* number of piecewise regions (or single region) through interpolating models for whole trajectory. The model generation steps are summarized as follows.

1. Simulate the full system with training input proposed in section IV.B and transfer data to MATLAB environment as explain in section IV.A.
2. Initialize order *n* for Chebyshev polynomials.
3. Generate Chebyshev interpolating knots using equations (3) or (4) according to input range.
4. Employ Newton divided difference method to obtain approximating polynomial in terms of state variable *x*, such that

$$f_c(x) \approx c_1x^{n-1} + c_2x^{n-2} + c_3x^{n-3} + ... + c_n^1 \qquad (9)$$

5. Traverse the full training trajectory while ensuring that relative error $relerror = \left|\frac{f(x) - f_c(x)}{f_c(x)}\right| < \delta$. δ is the relative error tolerance.
6. If *relerror* > δ, the order of polynomial increases by some predefined increment step γ. Value of γ is usually set in the range of one to five. The process repeats from step 2 to 6 until transient simulation data ends.

V. Evaluation of Chebyshev Polynomial Macromodels

A simplified nonlinear transmission line shown in fig. 3 [9] is employed to evaluate the algorithm and then compare it with Taylor polynomial based PWP implementations. The circuit consists of resistors, capacitors and diodes with constitutive equation $i_d = \exp(40v) - 1$ .All resistors and capacitors have unit value, i.e., *R*=*C*=1. The single input is the current source entering node 1, *u*(*t*) = *i*(*t*); the single output is the voltage at node 1, $y(t) = v_1(t)$.

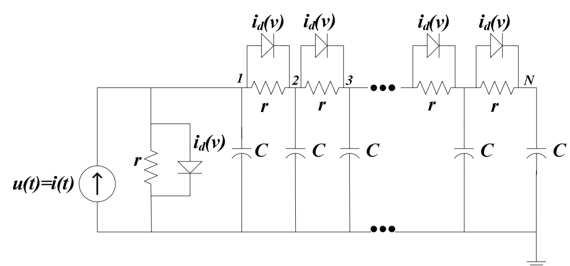

Fig. 3. Nonlinear Transmission Line [9]

Macromodel is trained using a combination of sinusoidals with different amplitudes, phases and frequencies. To evaluate the macromodel generated using Taylor and Chebyshev polynomials, a transient simulation of 200ms is run with input signal of *u*(*t*)=*i*(*t*)=2*sin*(*2π10t*), input current amplitude of ±2A and 10Hz frequency. In this implementation, we generate single Taylor and Chebyshev polynomial models representing nonlinear function *f(x)* for the whole trajectory. Figure 4 show the simulation results of Chebyshev and Taylor polynomial macromodels. Order of both polynomials is set to *10*.It is very clear from figure that output generated by Chebyshev macromodel matches exactly with original systems output. Whereas, there is lose in accuracy for Taylor polynomial macromodel.

To evaluate these macromodels for inputs far from training input, we run a transient simulation in which input is larger than the training input, i.e., $u(t)=2.6sin(2pi10t)$ with input current amplitude of $\pm 2.6A$. Order of both polynomials is 10. One can clearly observe from the figure 5 that accuracy of output voltage for Taylor macromodel is poor.

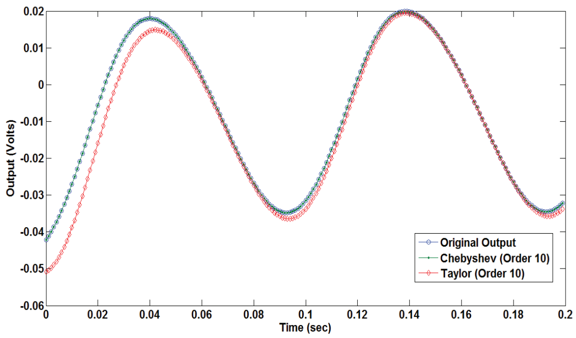

Fig. 4. Chabyshev and Taylor polynomial macromodels outputs for input of $i(t)=2sin(2\pi 10t)$

It is mentioned in section III.A that as input starts moving far from training trajectory the output of the Taylor polynomial macromodel starts diverging from the original solution. In this case, input is only *0.1A* far from training trajectory and there is a significant lost in accuracy for Taylor macromodel. On the other hand, Chebyshev macromodel output match with original output with very good accuracy.

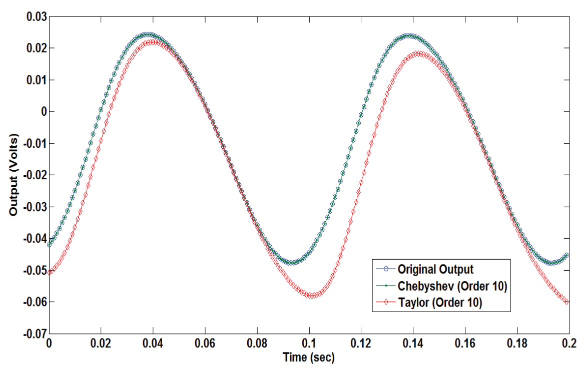

Fig. 5. Chebyshev and Taylor polynomial macromodels outputs for input of $i(t)=2.6sin(2\pi 10t)$

We also test the capability of chebyshev polynomial macromodel for the case when system input is far from training input. We run a transient simulation with input signal $u(t)=5.4sin(2pi10t)$, i.e., input current swings between $\pm 5.4A$, which means that input is 2.9A far from training input.

It is very clear from the result of figure 6 that even for the input far from training input, Chebyshev macromodel accuracy is reasonable. This proves that Chebyshev macromodels are efficiently able to interpolate the states that are not excited by training input.

We also test macromodels for two other inputs i.e., $i(t)$=exp(-t) and $i(t) = (cos(2\pi t/10)+1)/2$ [4]. The simulation results are shown in figure 7 and figure 8.

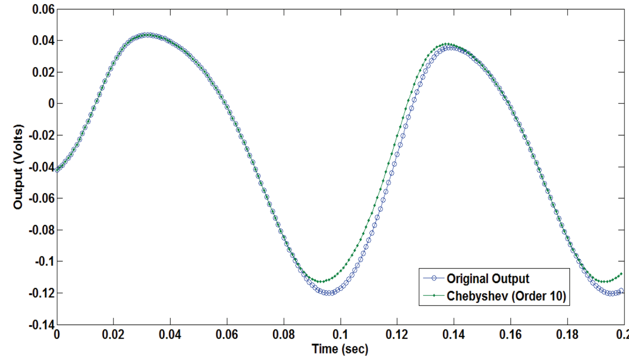

Fig. 6. Chebyshev polynomial macromodel output for input of $i(t)=5.4sin(2\pi 10t)$

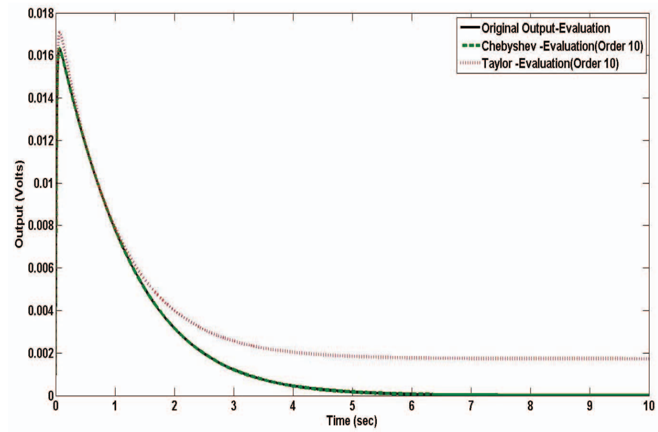

Fig. 7. Chebyshev and Taylor polynomial macromodels outputs for input of $i(t)$=exp($-t$)

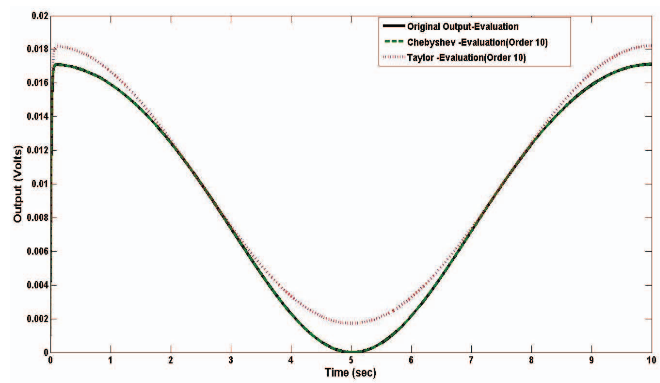

Fig. 8. Chebyshev and Taylor polynomial macromodels outputs for input of $i(t)= (cos(2\pi t/10)+1)/2$

It can be seen from simulation results in figure 8 and figure 9 that Taylor polynomial macromodel response (in red) diverge from original systems response. On the other hand Chebyshev-Newton polynomial macromodel response (in green) is almost indistinguishable from original system response (in black).

We also measure the simulation speed of both macromodels. To measure speed, we count the number of samples in which response of system becomes at steady state. Taylor polynomial macromodels stabilize after 56 samples whereas Chebyshev polynomial macromodels stabilize after 5 samples that are negligible. This proves that Chebyshev macromodels perform much faster than the Taylor macromodels.

VI. CONCLUSION

This paper presents a 2D scalability approach to improve the local approximation properties of TPWL macromodels through the utilization of Chebyshev interpolating polynomials. To scale the SS horizontally, we use single polynomial of order 10 for both Chebyshev and Taylor macromodels to cover the whole trajectory. Both achieve the good accuracy under the same conditions but Chebyshev macromodels run faster than Taylor macromodels. We also compare their capabilities to predict responses for the inputs far from training trajectory. Compared with Chebyshev, Taylor macromodels lose their accuracy significantly even when input is ±2.6A i.e. only ±0.1A far from training input current of ±2.5A. On the other hand Chebyshev macromodel accuracy is very good for input ±0.1A far from training. The upper bound of Chebyshev macromodel is also tested with the input current of ±*5.4A* that is ±2.9A far from training input swing ±2.5A.Chebyshev macromodel shows very reasonable accuracy for this input as well. Therefore, Chebyshev polynomials are able to scale trajectory models in 2D and achieve the goal of simulation speedup as compared to conventional Taylor polynomial macromodels.

In future, this macromodeling technique will be applied to more complex systems and implemented in high level language, i.e., VHDL-AMS to achieve speed up as at system level.

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