

Topology Optimization of Beam Structures with Various End and Loading Conditions

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Abstract. Topology optimization deals with optimum material (mass) distribution within a design domain to find optimal lay-out of a structure subjected to certain boundary and loading conditions. Topology optimization can be used to address conflicting requirements, such as light weight and high-strength/stiffness design. In this paper, a simulation program to analyze topology optimization of beam structures with seven different end conditions and three types of loads (single point load, two point loads and uniformly distributed load) is developed using MATLAB code adapted from Sigmund's 99 line topology optimization code. Furthermore, the program has been enhanced with a graphical user interface for ease of use. Using the developed system, it is possible to analyze the effect of different parameters.

Introduction

Structural optimization can be categorized into three different classes, namely size optimization, shape optimization and topology optimization [1]. Topology optimization encompasses the other two cases where the size, shape and topology are optimized to give the best possible design. The optimized design structure may have different size, shape and topology compared to the initial design domain depending on the load and boundary condition. Topology optimization deals with optimum material (mass) distribution within a design domain to find optimal lay-out of a structure subjected to certain boundary and loading conditions.

Topology optimization of structures has been an active research area since the late 1980s [2]. During this time, a number of methods or approaches have been proposed to address structural topology optimization such as homogenization based approach[3], power law approach [4, 5], ESO (Evolutionary Structural Optimization) [6], and level-set method [7]. Among these, the power-law or SIMP (Solid Isotropic Material with Penalization) approach and ESO are the most commonly used and referred methods in the literature.

In SIMP approach, the material properties are assumed to be constant within each element used to discretize the design domain, and the element relative densities are considered as variables. This approach models the material properties through relative material density which is being raised to a certain power and multiplied with material properties of solid material. This method is the most widely accepted and applied method including in some commercial software.

Evolutionary structural optimization approach, proposed by Xie and Steven[6] is an empirical concept in which a structure evolves towards optimum by slowly removing inefficient material i.e. material with lowest stress. However this method has difficulty in obtaining the global optimum. To address this difficulty, Bi-directional Evolutionary Structural Optimization (BESO) which is an extension of ESO was proposed, allowing the removal of inefficient material and addition of efficient material (with higher stress value than maximum stress) to the structure [8].

Application of optimization principles in today's competitive environments is essential specially in addressing conflicting requirements, such as light weight and high-strength/stiffness design. Beams are structures that can only carry transverse and bending loads perpendicular to the

longitudinal axis. Beams are widely applied in buildings, bridges and frames of mechanical systems. To be economical and safe, beam structures under specified boundary condition and applied load, are expected to be as stiff as possible with minimum material usage and weight.

In this paper, a simulation program is developed to analyze topology optimization of beam structures with different loading and end conditions using MATLAB code. The code has been adapted from Sigmund's 99 line topology optimization codes [9, 10]. A user friendly graphical user interface (GUI) has been developed for ease of use.

Mathematical Formulation

A typical optimization problem requires the formulation of objective function subjected certain constraints. Consider a statically loaded linearly elastic structure under a single loading condition, where the target is to get maximum stiffness with minimum material usage. In such cases, the typical objective function will be the minimization of compliance which is the inverse of stiffness. Compliance is defined as the dot product of force and displacement given by Eq. 1.

$$c = \vec{F} \cdot \vec{U} = F^T U. \quad (1)$$

Where c is the compliance

\vec{F} is the force vector

\vec{U} is the global displacement vector

Both force and displacement vectors are column matrices. The definition of compliance can be further refined by introducing elastic equilibrium equation given by Eq. 2.

$$F = KU. \quad (2)$$

Where K represents the global stiffness matrix.

The Young's modulus for each element, e , is defined as [10] :

$$E_e(x_e) = E_{\min} + x_e^p (E_0 - E_{\min}), \quad x_e \in [0,1]. \quad (3)$$

Where E_e represents the Young's modulus for element e ,

E_{\min} represents a very small value of Young's modulus for void region to avoid singularity,

x_e is elements relative density,

p is the penalization, and

E_0 represents the Young's modulus for solid material.

The global stiffness matrix, K , in terms of the Young's modulus is given by Eq. 4.

$$K = \sum_{e=1}^N E_e(x_e) k_0 = \sum_{e=1}^N [E_{\min} + x_e^p (E_0 - E_{\min})] k_0. \quad (4)$$

Where N is the number of elements, and

k_0 is the elemental stiffness matrix.

From Eq. 1, Eq. 2, and Eq. 4 the objective function of the form given by Eq. 5 can be obtained.

$$\min_x : c(x) = \sum_{e=1}^N E_e(x_e) u_e^T k_0 u_e. \quad (5)$$

After formulating the objective function, the constraints confining the objective function should be defined. Firstly, the objective function should be subjected to volume fraction constraint as the resulting structure will have less volume compared to the initial design domain. Let $V(x)$ be the volume of the resulting structure, V_0 be the volume of the initial design domain and f be the volume fraction. The volume fraction is given by:

$$\frac{V(x)}{V_0} = f. \quad (6)$$

Apart from volume fraction constraint, the objective function should also comply with the elastic equilibrium equation mentioned in Eq. 2 above. Next, lower and upper bounds for the element

relative density should also be defined. The upper bound for the element relative density is set at one, which represents solid material, and the lower bound is set to zero representing void element. In the earlier versions of SIMP method, the lower bound was set to a very small number close to zero but bigger than zero to avoid singularity. This issue of singularity has been accounted in the definition of Young's modulus in Eq. 3. Hence, the constraint for the design variable, x is given by:

$$0 \leq x \leq 1. \quad (7)$$

Combining the objective function with all these constraints, the topology optimization problem formulation is given by:

$$\begin{aligned} \min_x : c(x) &= \sum_{e=1}^N E_e(x_e) u_e^T k_0 u_e \\ \text{subject to : } & \frac{V(x)}{V_0} = f \\ & : KU = F \\ & : 0 \leq x \leq 1 \end{aligned} \quad (8)$$

Results and Discussion

Based on the mathematical formulation given in the previous section, existing MATLAB codes [9, 10] have been adapted to analyze topology optimization of beam structures with different end and loadings conditions. Furthermore, a user friendly graphical user interface (GUI) has been developed for ease of use. Fig. 1 shows the flow chart of the program developed. The program is compiled to self-executable form which does not require Matlab software to be installed on the machine for execution.

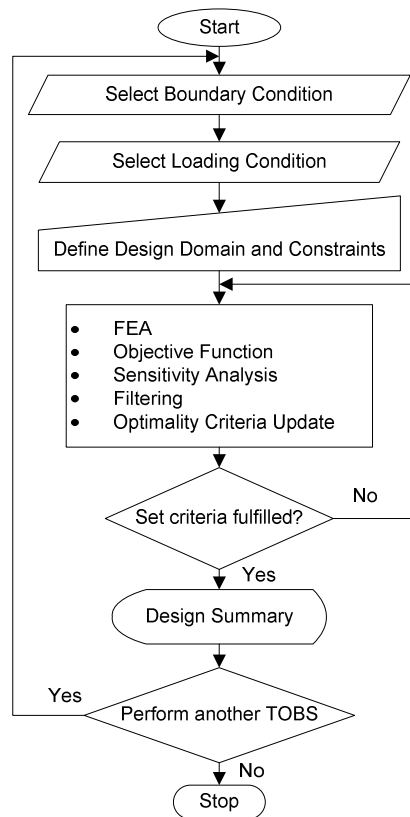


Fig. 1 Flow chart of the developed program

Boundary conditions: The program supports seven different boundary conditions, namely: simply-supported beams, cantilevered beams, one end fixed the other simply supported, both ends fixed, overhanging (two cases), and one end with two fixed support the other end with roller support. Through the GUI the user can select the required boundary condition as shown in Fig. 2.

Loading conditions: After the user selects the boundary condition and click next button on Fig. 2, the user will select the loading condition for the design as shown in Fig. 3. As can be seen in Fig. 3, the load types are basically categorized into two broad classes namely point load and uniformly distributed load. Under point load selection, the user can apply single load or multiple loads.

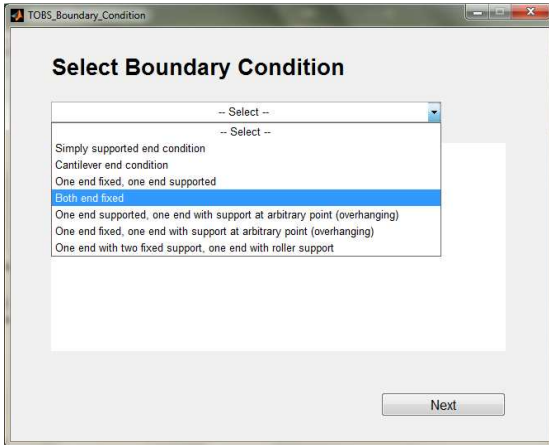


Fig. 2 Screenshot of boundary condition selection window

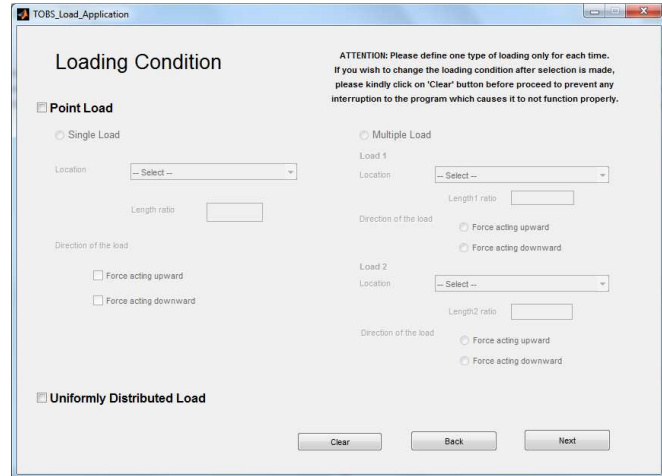


Fig. 3 Screenshot of load application window

User Data Input: After boundary and loading conditions have been selected, the user will give additional parameters required for the finite element analysis and optimization through user data input window (Fig. 4). These parameters include: number of elements in x and y directions for FEM analyses, volume fraction, penalization power which is usually taken to be less than or equal to 3 as proposed in SIMP method, filter radius is normally taken to be 3 or the size of several elements, and filter type where either sensitivity or density filter are chosen .

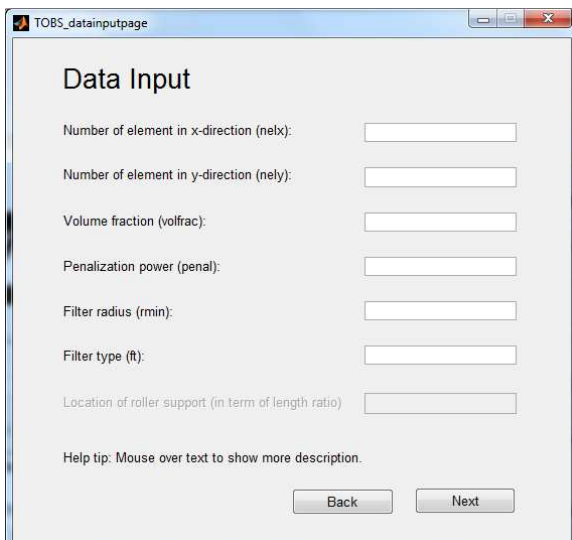


Fig. 4 Screenshot of data input window

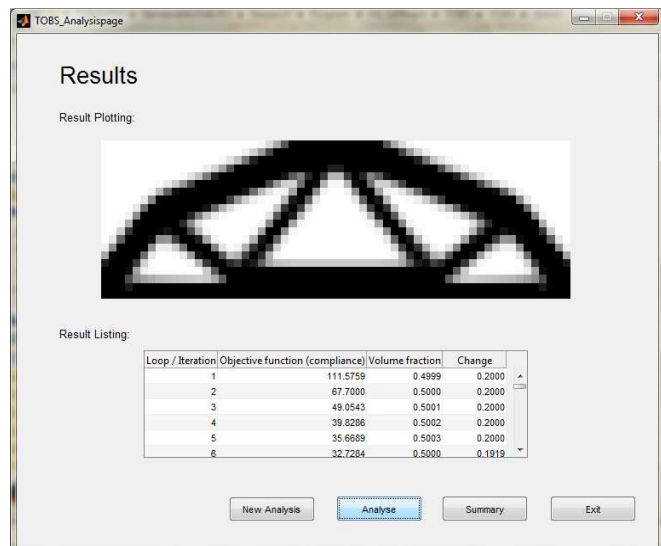


Fig. 5 Screenshot of results page

Once all the required inputs are given, the system can analyze and provide the optimum topology together with the number of iterations, the compliance and plot as shown in Fig. 5. The summary button brings the design summary including the user inputs for the analysis on new window (not shown for brevity). The developed software package can be used to analyze the effect of different parameters such as effect of mesh size on compliance.

The result shown in Fig. 5 is obtained by considering a simply supported beam with the following parameters: a point downward load at the center of the beam, number of elements in x

and y directions are 60 and 20 respectively, volume fraction is 0.5, penalization power is 3, filter radius of 1.5 and sensitivity type filter is used.

Conclusions

Topology optimization is a method aimed at distributing material within a design domain to produce an optimal structure. This optimal structure must comply with the specified boundary conditions and able to sustain the loading condition given. In this paper, topology optimization of beam structures has been demonstrated using finite element based topology optimization principle. Beam structures with seven different end conditions can be analyzed using the developed TOBS software. Single point load, two point load and uniformly distributed loading conditions were implemented. The effect of different parameters can easily be analyzed by using the developed system.

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