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## Frequency Domain Analysis of Truss Spar Platform

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### ABSTRACT

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*Spar is a type of deepwater floating type of platform used in ultra deep water. Malaysia has recently installed its first spar at Kikeh field near Sabah and is the first one ever installed outside the Gulf of Mexico. In this study, dynamic analysis of a typical truss spar in frequency domain has been conducted and the motion responses in surge, heave and pitch have been evaluated. The truss spar has been modeled as a rigid body with three degrees of freedom (i.e. surge, heave, pitch) at its centre of gravity, connected to the sea floor by ten multi-component catenary mooring lines attached to the truss spar at the fairleads. Frequency domain analysis has been performed by choosing a suitable wave spectrum model to represent an appropriate density distribution of sea water at the site under consideration. The motion response spectra have been determined based on the wave spectrum for each of the motion and the motion response profiles are evaluated from the spectra. The maximum amplitudes obtained were 0.25 m for surge, 0.02 m for heave and 0,013 radians for pitch. These results have the same trend but lower amplitude compared to the responses obtained by time domain dynamic analysis.*

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**Keywords:** *Kikeh truss spar, Dynamic response, Frequency domain, Wave spectrum.*

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## **1. INTRODUCTION**

Spar is a large, deep draft and cylindrical floating caisson designed to support drilling, production operations and storage of oil in Oil and Gas industry. In 2007, Malaysia has installed its first spar at Kikeh field located in 1330m water depth offshore Sabah and is also the first one ever installed outside the Gulf of Mexico. Historically, the applications of spars were as marker buoys to gather oceanographic data for research purposes and as storage containers for oil like floating instrument platform (FLIP) and Brent spar (Argawal & Jain, 2003)[1]. In recent years, there has been an increasing interest in utilizing spar technology for deep and ultra-deep production platforms. According to Mekha et al., (1995, 1996)[9-10], the spar can be installed regardless of water depth, number of wells and deck load as its heave natural period is dependant only on the draft of the spar.

The basic parts of a spar include deck, hard tank, midsection (caisson hull or truss structure) and soft tank. Up to date, three types of production spars have been built. They are Classic, Truss and Cell spars. The world first production spar, Neptune Spar was based on Classic Spar design. In the later stage, the spar design evolved into truss section by replacing the midsection with truss structure (Truss Spar) and multiple ring stiffened tubes or ‘cells’ to form the spar hull (Cell Spar). In a review of the advantages of classic spar over other floating platforms, Downie et al. (2000) and Sadeghi et al., (2004)[7,11] identified five distinctive characteristics of classic spar including viable option to use in severe environmental condition, low motion, low cost, structural simplicity and a protected centerwell. Several studies have revealed that the truss spar is better than classic spar in that it offers lower cost, lower weight, shorter construction duration, dampened heave motion, less drag provided by the truss and reduced overall mooring system loads in high current environment (Chakrabarti, 2005; Downie et al.; Sadeghi et al.; Wang et al., 2001,2002)[5,7,11,13-14].

Results of numerous hydrodynamics analysis and motion response predictions technique have been developed and introduced in various technical papers (Anam et al., 2003; Argawal & Jain; Burke&Tighe, 1972; Downie et al; Mekha et al; Sadeghi et al.; Spanos et al., 2005)[1,3-4,7,9-12]. Generally, there are two basic approaches used in performing dynamic analysis of floating structures; frequency domain and time domain analysis. The frequency domain analysis is less time consuming and simpler compared to time domain analysis because the response estimation can be carried out using wave spectrum method. However, there is a limitation for the frequency domain analysis that all nonlinearities in the equation of motion are replaced by the linear approximations where it will lead to low accuracy and error in response prediction. The nonlinearities are in fluid drag force, mooring line force, viscous damping and stiffness of the system for different motions consideration. In this paper, a typical truss spar is selected for frequency domain analysis. The purpose of this study is to gain general understanding on truss spar responses subjected to random waves using simpler dynamic analysis approach.

## **2. STRUCTURAL MODEL**

The truss spar was modeled as a rigid body with three degrees-of-freedom (i.e. surge, heave, pitch) at its centre of gravity, connected to the sea floor by ten (10) multi-component catenary mooring lines, which are attached to the truss spar at the fairleads. The ten catenary mooring lines keep the spar stable at the location. The centre of gravity of truss spar is always above the centre of buoyancy to provide inherently stable design for truss spar. Figure 1 shows a typical offshore truss spar platform.

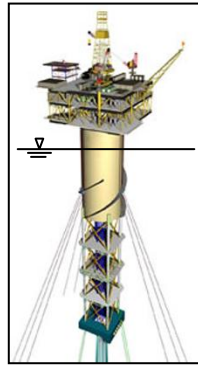


Figure 1. Typical offshore truss spar platform

The truss spar consists of topside located above the Hard Tank. It has a cylindrical upper hull (Hard Tank) with a square center well, a jacket-type middle-section truss (three bays) with two heave plates and a soft tank (keel tank) at the keel. The mooring system is a ten-leg deep water taut catenary system with chain jacks installed on the hull top level and fairleads installed on the bottom of Hard Tank. The principle dimensions of the typical spar hull and wave data are given in Table 1.

Table 1. Dimensions of truss spar and wave data

Total truss spar hull length	141 m
Total draft	131 m
Hard tank diameter	32.3 m
Hard tank freeboard	11 m
Hard tank length	67 m
Soft tank length	11 m
Total truss length	64 m
No. of heave plate	2
Heave plate size	32.3 m <sup>2</sup>
Truss leg spacing	22.86 m
Vertical truss member diameter	1.60 m
Diagonal truss member diameter	0.75 m
Weight of the truss spar	51000 t
Distance of centre of gravity from keel	81.72 m
Structural damping ratio	0.1
Wave period	13.1 sec
Wave height	12.0 m
Water depth	1330 m
Drag coefficient ( $C_d$ )	0.7
Inertia coefficient ( $C_m$ )	2.0

### 3. METHODOLOGY

#### 3.1 Coordinate System

The platform global axis system used for the calculation of wave forces and moment is shown in Figure 2. All locations are specified based on this coordinate system. The origin was taken at the Longitudinal/Transverse Centerline at the top of Hard Tank (below the freeboard) with Y axis positive up. The longitudinal axis (X-axis) was along platform East-West with positive towards

East. The transverse (Z-axis) direction was along platform North-South with positive towards North.

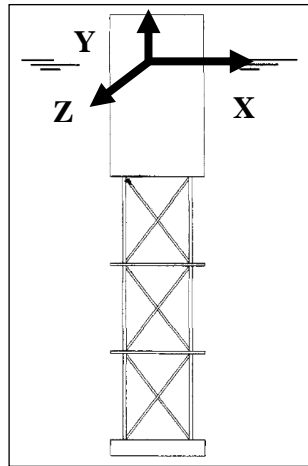


Figure 2. Axis Coordinate System

### 3.2 Wave Forces and Moment Calculation

The wave force acting on an offshore structure is usually the most important of all environmental loadings. The wave forces are developed because of the motion of water particles hitting the structure with velocities and accelerations. The calculation of wave loads on the truss spar is based on Morison's equation applied in conjunction with linear wave theory. Truss spar is considered as hydro-dynamically transparent with no significant influence on the wave field. It is because the ratio of the truss spar diameter to wave length is small ( $D/L < 0.2$ , where  $D$  is the structure diameter and  $L$  is the wave length).

Morison's equation expresses the wave force as the sum of an inertia force proportional to the particle acceleration and a non-linear drag force proportional to the square of the particle velocity:

$$F = F_I + F_D \quad (1)$$

$$F = C_M \frac{\rho \pi D^2}{4} u' + C_D \frac{\rho D}{2} |u| u \quad (2)$$

where,  $F$ , wave force per unit length on a circular cylinder;  $u$  and  $|u|$ , water particle velocity normal to the cylinder, calculated with the selected wave theory at the cylinder axis;  $u'$ , water particle acceleration normal to the cylinder, calculated with the selected wave theory at the cylinder axis;  $\rho$ , sea water density;  $D$ , member diameter; and  $C_d$ ,  $C_m$ , drag and inertia coefficients, respectively.

By using linear wave theory, with a wave height and wave period chosen according to the location of the structure, the corresponding horizontal and vertical components of wave particle velocity and acceleration were determined. The kinematics of the wave water were determined by the following equations:

$$\text{Horizontal Water Particle Velocity,} \quad u = \frac{\pi H}{T} \frac{\cosh ks}{\sinh kd} \cos \theta \quad (3)$$

$$\text{Vertical Water Particle Velocity,} \quad v = \frac{\pi H}{T} \frac{\sinh ks}{\sinh kd} \sin \theta \quad (4)$$

Horizontal Water Particle Acceleration, 
$$u' = \frac{2\pi^2 H}{T^2} \frac{\cosh ks}{\sinh kd} \sin \theta \tag{5}$$

Vertical Water Particle Acceleration, 
$$v' = -\frac{2\pi^2 H}{T^2} \frac{\sinh ks}{\sinh kd} \cos \theta \tag{6}$$

where,  $s = y + d$ ;  $\theta = kx - \omega t$ ;  $k$ , wave number ( $2\pi / L$ );  $\omega$ , natural frequency, ( $2\pi / T$ );  $T$ , wave period;  $y$ , height of the point of evaluation of water particle kinematics;  $x$ , point of evaluation of water particle kinematics from the origin in the horizontal direction;  $t$ , time instant at which water particle kinematics is evaluated;  $L$ , wave length;  $H$ , wave height; and  $d$ , water depth.

Determination of wave forces on the truss spar was divided into four sections including hard tank, truss section – level 1, truss section – level 2 and truss section – level 3. The wave was assumed to hit the structure in X direction and the entire truss spar structure was considered vertical in place, no inclination in Y axis. Heave plates and soft tank were not included in the wave forces calculation because their sizes and orientation contributed only insignificant wave forces.

### 3.3 Frequency Domain Analysis

Frequency domain analysis was performed first by choosing a suitable wave spectrum model to represent an appropriate density distribution of sea wave at the site under consideration. The analysis was performed in the frequency domain. Secondly, the motion-response spectrum was determined based on the wave spectrum for the responses in surge, heave and pitch degrees of freedom. Finally, the motion response profile was simulated from the motion-response spectrum.

#### 3.3.1 Wave Spectrum

The energy density spectrum: Pierson-Moskowitz (P-M) spectrum model was used for the frequency domain analysis. The expression for the P-M spectrum in terms of cyclic frequency  $f$  ( $\omega/2\pi$ ) may be written as

$$S(f) = \frac{\alpha g^2}{(2\pi)^4} f^{-5} \exp \left[ -1.25 \left( \frac{f}{f_0} \right)^{-4} \right] \tag{7}$$

where,  $\alpha = 0.0081$  and peak frequency,  $f_0(\omega_0 / 2\pi)$

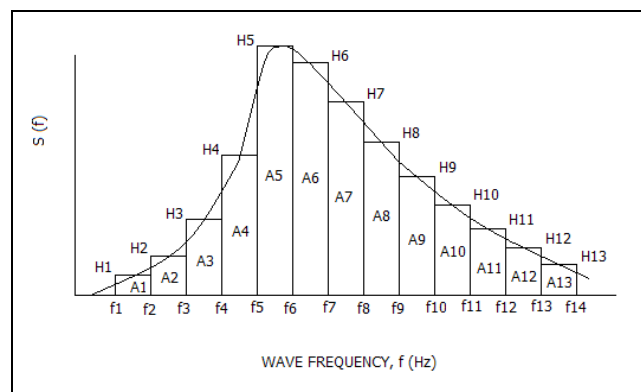


Figure 3. Energy contribution at a range of frequencies of an energy density spectrum

## Frequency Domain Dynamics Analysis of Truss Spar Platform

For this spectrum, the relationship between the peak frequency and the significant height for the wave was as follows:

$$\omega_o = 0.161g / H_s \quad (8)$$

Referring to Figure 3, at frequency,  $f_1$ , the energy density was  $S(f_1)$ . The weight height at this frequency was obtained as follows:

$$H(f_1) = 2\sqrt{2(f_1)\Delta f} \quad (9)$$

The time history of the wave profile was determined from:

$$\eta(x,t) = \sum_{n=1}^N \frac{H(n)}{2} \cos[k(n)x - 2\pi f(n)t + \varepsilon(n)] \quad (10)$$

where,  $x$  was location of evaluation of wave profile from the origin in the horizontal direction;  $t$  was the time instant at which wave profile was evaluated and was incremented; wave number  $k(n)$ ; wave length  $L(n)$  corresponded to the wave length for  $n$ th frequency  $f(n)$ ; wave height  $H(n)$  was computed from Equation (9) for  $n$ th frequency; and the  $n$  was the total number of frequency band of width  $\Delta f$ , dividing the total energy density as shown in Figure 3.

### 3.3.2 Motion Response Spectrum

The responses of the truss spar towards the motions of surge, heave and pitch were calculated by multiplication of the wave energy spectrum (Equation 7) with the square of RAO function to evaluate the response spectrum value at particular frequency. The expression of motion-response spectrum may be written in the following two (2) forms:

$$S_x(f) = [RAO(\omega)]^2 S(f) \quad (11)$$

$$S_x(f) = \left[ \frac{F_I / (H/2)}{[K - m\omega^2]^2 + (C\omega)^2} \right]^{1/2} S(f) \quad (12)$$

where, RAO was amplitude of response per unit wave amplitude;  $F_I$ , was inertia force;  $K$  was stiffness of the structure associated with different type of motion;  $m$  was summation of mass and added mass of the structure associated with different type of motion;  $C$  was structural damping ratio;  $H$  was wave height corresponding to particular frequency; and  $\omega$  was natural frequency corresponding to particular frequency.

### 3.3.3 Stimulation of Motion Response Profile from Spectrum

From the resulting motion-response spectrum, the expected response (time history) profile in a given time interval was easily deduced. Equation (10) was used to determine the response profiles.

## 4. NUMERICAL RESULTS AND DISCUSSIONS

### 4.1 Wave Forces and Moment

Wave forces calculated using Morison's equation ( $C_m = 2.0$  and  $C_D = 0.7$ ) at different sections of the truss spar is summarized in Table 2. The wave force was assumed to act at the origin,  $x$  ( $x = 0$  m) and when the time,  $t$  was 0 second. According to the wave force distribution summary presented in Table 2, almost all of the wave force (in  $X$  direction) was taken by the hard tank and the truss section experienced insignificant wave force.

Table 2: Wave force distribution on the truss spar

Sections	$F_x$ (kN)	$F_y$ (kN)	$F_z$ (kN)
<b>Hard Tank</b>	1889.00	0.00	0.00
<b>Truss 1</b>	-101.66	-1.31	0.00
<b>Truss 2</b>	-43.38	-0.62	0.00
<b>Truss 3</b>	-26.70	-0.47	0.00
<b>Total Force</b>	<b>1717.00</b>	<b>-2.00</b>	<b>0.00</b>

Table 3 summarizes the moment distribution on the four (4) sections of truss spar and the total moment experience by the entire truss spar. Due to large wave force acting on the hard tank, large moment (in clockwise direction) was also induced at hard tank. The direction of moment was controlled by wave forces acting on the hard tank.

Table 3. Moment distribution on the truss spar

Sections	$M_x$ (kNm)	$M_y$ (kNm)	$M_z$ (kNm)
<b>Hard Tank</b>	61174.98	0.00	0.00
<b>Truss 1</b>	42866.72	-12.42	0.00
<b>Truss 2</b>	41087.61	-12.42	0.00
<b>Truss 3</b>	40213.90	-10.70	0.00
<b>Total</b>	<b>-185343.21</b>	<b>-35.54</b>	<b>0.00</b>

## 4.2 Wave Spectrum

The wave energy density spectrum,  $S(f)$  was determined based on the equation (7) and the significant height was obtained from a wave record of significant height 6.3m. The P-M spectrum was drawn range from the frequencies of 0.005 Hz to 0.250 Hz with a frequency increment of 0.01 Hz (Figure 4) and the corresponding wave height from each frequency in the range was obtained. From the calculated wave height and equation (10), the time history of the wave profile ( $t = 0$ second to  $t = 200$ seconds) in front of the truss spar at  $x = 0$ m (initial position) was computed and a random phase in the range of  $(0, 2\pi)$  was assigned to a random number generator,  $R_N$  to retain randomness of the time history. The stimulated wave profile is shown in Figure 5.

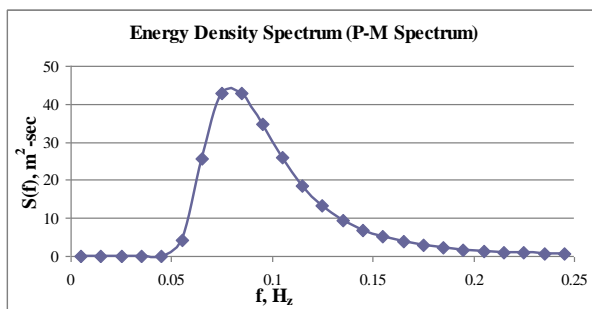
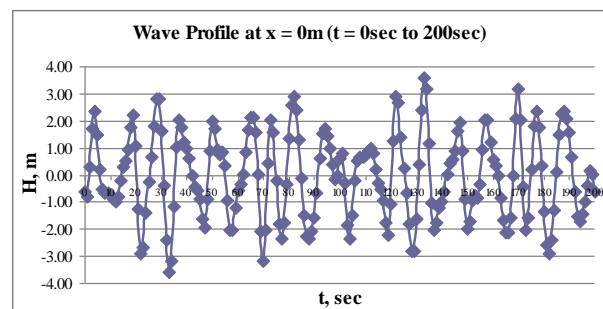
Figure 4. P-M spectrum for  $H_s = 6.3$ m

Figure 5. Simulated wave profile

Based on the wave spectrum illustrated in Figure 4, the wave energy at frequency 0.08Hz was highest among all and from the stimulated wave profile (Figure 5), the highest wave height in front of truss spar was approximately 3.5m.

### 4.3 Motion Response Spectrum

In determining the motion-response spectrum, three type of motions: surge, heave and pitch were chosen to analysis the response of truss spar towards these three motions. As stated in Section 3.3.2, the motion-response spectrum of surge, heave and pitch were determined based on equation (12), structure damping ratio 10%, and the P-M model of  $H_s=6.3\text{m}$ . For the computation of RAO, the wave train was considered as random and frequencies (0.05 Hz to 0.25 Hz) in the wave train were chosen to cover the entire range of frequencies of the wave spectrum. RAOs for all the motions were multiplied with P-M spectrum to finally obtain the response spectrum. Figures 6, 7 and 8 illustrate the response spectrum for surge, heave and pitch, and all these response spectra have a maximum peak value close to the wave spectrum peak.

### 4.4 Response of Truss Spar on Surge, Heave and Pitch Motions

The calculated responses of the structure are shown in Figures 9, 10 and 11. The maximum amplitudes of the three motion responses were as follows:

- Surge: 0.25 m
- Heave: 0.02 m
- Pitch: 0.013 radians

The predicted responses of truss spar were only approximate due to the following reasons:

- There is a limitation of frequency domain technique that all nonlinearities in the equations of motion (Equation 12) were replaced by linear approximations.
- The actual stiffness of mooring lines was not known and thus the computation of stiffness was simplified by using static equilibrium conditions.
- The mass moments of inertia were calculated based on assumed distribution of masses.

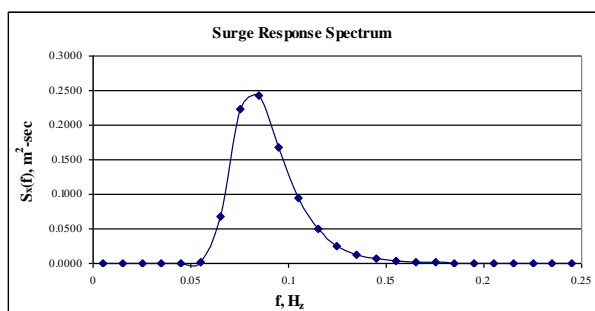


Figure 6. Surge Response Spectrum

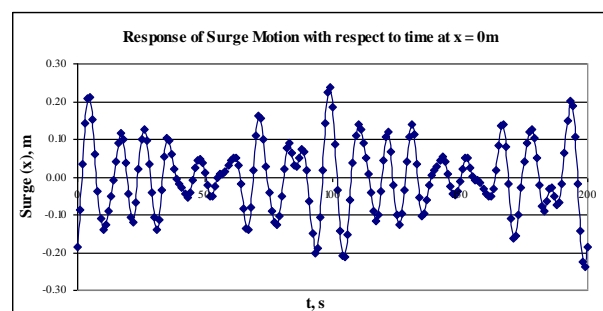


Figure 9. Simulated surge profile

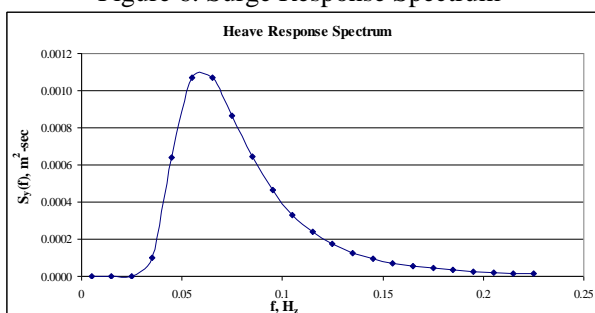


Figure 7. Heave Response Spectrum

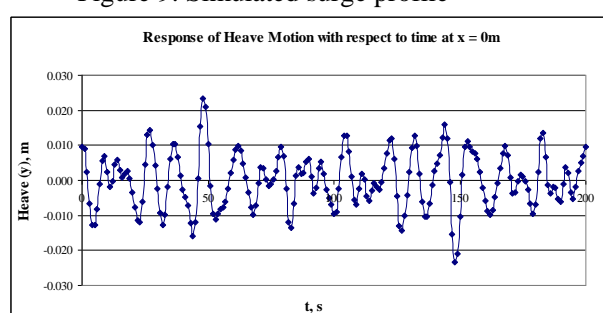


Figure 10. Simulated heave profile



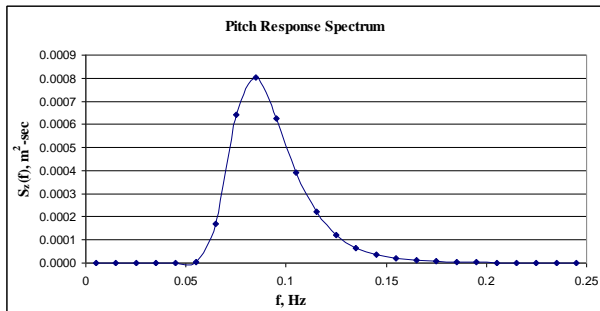


Figure 8. Pitch Response Spectrum

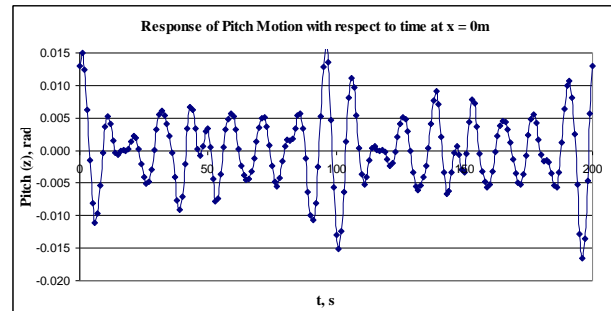


Figure 11. Simulated pitch profile

## 5. CONCLUSIONS

1. The developed frequency domain dynamic analysis of a typical spar has been able to predict the responses in surge, heave and pitch degrees of freedom when the spar was subjected to a random wave developed from Pierson-Moskowitz spectrum.
2. The maximum amplitudes obtained are 0.25 m for surge, 0.02 m for heave and 0.013 radians for pitch. The predictions using frequency domain are not very accurate as it can not take the nonlinearities into account. However the responses followed the same trend of the applied wave as shown by time domain results in literature.
3. The results of this frequency domain analysis can be very useful for the preliminary design of spar and its components.

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